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Internship research project: Implementing advanced routing in the SPHY model using the convection-diffusion equation

Author:
Femke Jansen

Supervisors

FutureWater:
Wilco Terink MSc.

Wageningen University:
Drs. Paul Torfs

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Abstract

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Hydrological models, such as the Spatial Processes in HYdrology (SPHY) model, are often used as a tool to simulate and predict discharge dynamics by modelling hydrological processes in catchments. This can be used to address water-related challenges in the area. To accurately simulate discharge in a channel it is important to implement streamflow routing in the model, which describes the transport of water through an open-channel network. An advanced routing procedure using the convection-diffusion (cD) equation has been implemented in the SPHY model. This cD-routing module has been set up outside the PCRaster environment which is used by the SPHY model. This provides the opportunity to run the routing module with a different temporal and spatial resolution than the simulation of the SPHY model itself, contrary to the currently implemented simple routing scheme. The cD-routing procedure has been tested in the Tamakoshi river basin in Nepal. After calibration of the wave celerity (c) and the diffusion coefficient (D) the results obtained are comparable to the currently implemented routing scheme in terms of model performance. NS values of 0.80 and 0.72 were obtained for the two observation stations Busti and Rasnalu respectively. Computational efforts are comparable for the current simple routing scheme and for the new implemented cD-routing scheme.

Keywords: SPHY, routing, convection-diffusion, numerical discretization, Tamakoshi, Nepal

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1. Introduction

Water shortage, as well as water excess, increase food and drinking water insecurity and additionally may induce ecological threats. Additionally, hydropower facilities depend on a constant water supply to produce enough energy. With the intention to develop hydropower in a region, an understanding of climate change impacts on streamflow and its uncertainty is crucial, as well as to assess the changing probabilities and magnitudes of extreme events. These emerging challenging situations are expected to increase as a consequence of climate variability and change (Rockström et al., 2012; Vörösmarty et al., 2000; Terink et al., 2015). Adequate water supply is essential for the well-being of humans and the environment.

Hydrological models, such as the Spatial Processes in HYdrology (SPHY) model (Terink et al., 2015), are often used as a tool to simulate and predict discharge dynamics by modelling hydrological processes in catchments. This can be used to address water-related challenges in the area (Terink et al., 2015; Wagener and Wheater, 2006; Pechlivanidis et al., 2011). To accurately simulate discharge in a channel it is important to implement streamflow routing in the model, which describes the transport of water through an open-channel network by simulating the propagation of the magnitude, volume and temporal pattern of the flow (Terink et al., 2015; Fread, 1985). Routing procedures create the possibility to describe the hydrograph along the channel network, which provides information about the changes in water flow, for instance the attenuation (diffusion effect) of peak discharges (Ramírez, 2000; O'Sullivan et al., 2012). In the lower areas of a mountainous catchment that are prone to flooding, this attenuating effect can reduce the effects of a flood. Therefore assessment of this effect can support decision makers in defining strategies (Montaldo et al., 2004).

1.1 Problem description

Various types of methods have been developed to apply routing, which can roughly be subdivided into: i) hydrological routing and ii) hydraulic routing. Currently there are two routing schemes included in the SPHY model (Terink et al., 2015) that can be assigned to hydrological routing. The first option is a simple flow accumulation routing scheme, in which for each cell the accumulated amount of water flowing out of the cell into its neighbouring cell downstream is calculated, which thus comprises the amount of water in the cell itself plus the amount of water in the cells upstream. The second option is the fractional accumulation flux routing scheme, which is used in the presence of lakes or reservoirs. In this case part of the water volume stored will become available for routing depending on the actual lake/reservoir storage. Subsequently, this flux will follow the simple flow accumulation routing scheme. A flow recession coefficient (kx) has been implemented in SPHY to account for flow delay, which is needed to prevent all the specific runoff generated within the catchment on one day to end up at the most downstream part of the catchment on that same day.

The current implemented routing schemes in SPHY have proven its applicability under various conditions (Terink et al., 2015). However, it has also shown to favour either quick or slow flow routes, reflected in simulating either simulate peak flows or base flow accurately (p.c. W. Terink), depending on catchment characteristics. Introducing an advanced routing scheme could improve streamflow simulations, which is necessary to address water-related challenges in an area.

1.2 Objectives and research questions

The objective of this research project is to explore options and to implement an advanced routing procedure to improve the current routing scheme and therefore streamflow simulations in the SPHY model.

From this objective the following research question has been formulated:

How can the routing procedure and therefore streamflow simulations of the SPHY model be improved?

In order to answer this main research question the following sub-questions are identified:

1. How is the routing procedure currently implemented in the SPHY model?
2. What channel and catchment characteristics are typically available in projects where the SPHY model is applied?
3. What are potential routing methods, considering available data, and how can these be implemented in the SPHY model?
4. What is the effect of using these routing methods on model performance in terms of streamflow simulations and on model computation time?

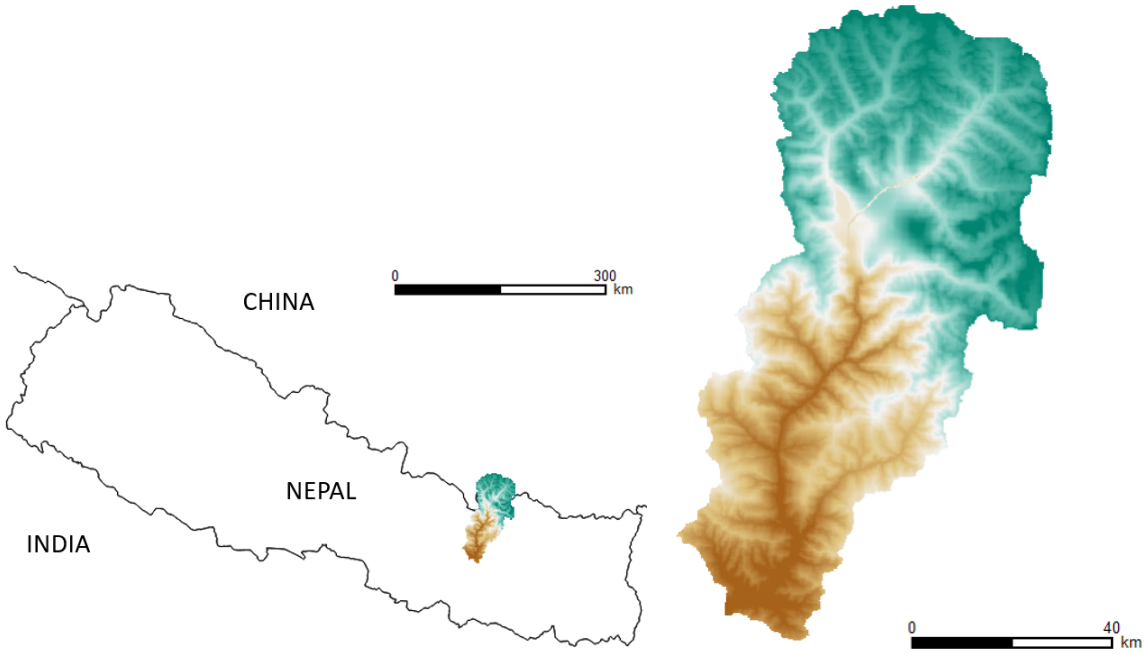
1.3 Structure of report

A description of the research area will be provided first (Chapter 2), followed by a brief description of the SPHY model (Chapter 3). Subsequently an overview is provided of potential routing methods (Chapter 4) and in chapter 5 several numerical approximation methods are explained. The implementation of the advanced routing scheme and the accompanying programming code is presented in chapter 6. The main results are discussed in chapter 7, and the report ends with the conclusion (Chapter 8) and recommendations (Chapter 9).

2. Application example

This research focuses on the Tamakoshi River Basin in Nepal where FutureWater is performing hydrological research. The SPHY model with its current routing module has already been set up and calibrated for this area. This facilitates comparison of the results with the newly implemented routing module. In this area catchment response and routing is dependent on natural processes without (too much) human intervention. The latter is highly likely to be of influence on the routing when considering a catchment in for instance The Netherlands, which could require additional components to be implemented in the routing procedure. This lies beyond the scope of this internship research project.

The Tamakoshi river basin is located in the north-east of Nepal and lies partly in China and covers an area of 2926 km^2. The glaciated basin lies between 27°37′42″N to 28°19′23″N latitude and 86°0′9″E to 86°34′12″E longitude in the southern slope of eastern Hindu Kush Himalayan region. Elevation roughly ranges from 850 masl to 7300 masl. On average 20% of the area is estimated to be covered with snow and around 80 glaciers are present in the river basin covering a total area of 110.00 km^2 (Khadka et al., 2014).



(a) Location map of Tamakoshi basin in Nepal **(b)** Digital Elevation Map (DEM)

Figure 2.1: Tamakoshi river basin

3. SPHY model

The Spatial Processes in HYdrology (SPHY) model developed by FutureWater is a spatially distributed leaky bucket type of model that includes the main terrestrial hydrological and glacier processes and has a flexible spatial resolution (Terink et al., 2015). The SPHY model is written in the Python programming language and makes use of the PCRaster (Karssenberg et al., 2001) dynamic modelling framework. The model code has been made freely available (version 2.1) (FutureWater, 2015). During this internship research project a yet to be released version of the model with improved glacier module has been used. Below a description of the model based on Terink et al. (2015) will be provided.

3.1 Model structure

SPHY is a spatially distributed leaky bucket type of model and is raster based. This means that changes in storage and fluxes are simulated on a cell-by-cell basis, which provides the opportunity to evaluate these changes over time and space. A schematic overview of the model structure is provided in figure 3.1. The soil structure consists of two upper soil reservoirs and a third groundwater reservoir, which complies with the VIC model structure. Drainage from these reservoirs occurs through surface runoff, lateral flow and baseflow respectively. The sum of these components, together with snowmelt and glacier melt if present, is the cell-specific runoff, which forms the water volume available for routing.

Precipitation that falls on a grid cell may be either classified as rain or snow, depending on temperature. Part of the precipitation will be intercepted and evaporated from the land surface, while another part of the liquid precipitation will become surface runoff or will infiltrate into the soil reservoir. There the remainder of the water that does not evaporate, either contributes to river discharge through lateral flow from the first soil layer, or as baseflow after it has percolated to the groundwater. Precipitation in the form of snow contributes to the snow storage, of which the balance is updated accordingly to the simulation of snowmelt and snow accumulation amounts.

Some of the hydrological processes incorporated in the model are contained in modules that can be switched on/off according to the relevance of these processes in the area of interest. This enables to reduce model run time and decrease the number of required input data. The decision on which modules to include in the modeling framework requires knowledge about the catchment characteristics and its relevant processes. The modules present are: glaciers, snow, groundwater, dynamic vegetation, simple routing, and lake/reservoir routing. A detailed description of all hydrological processes and modules incorporated in the SPHY model and how they are implemented can be found in Terink et al. (2015). A more detailed description of the routing procedures as currently implemented in the model will be given in the next section.

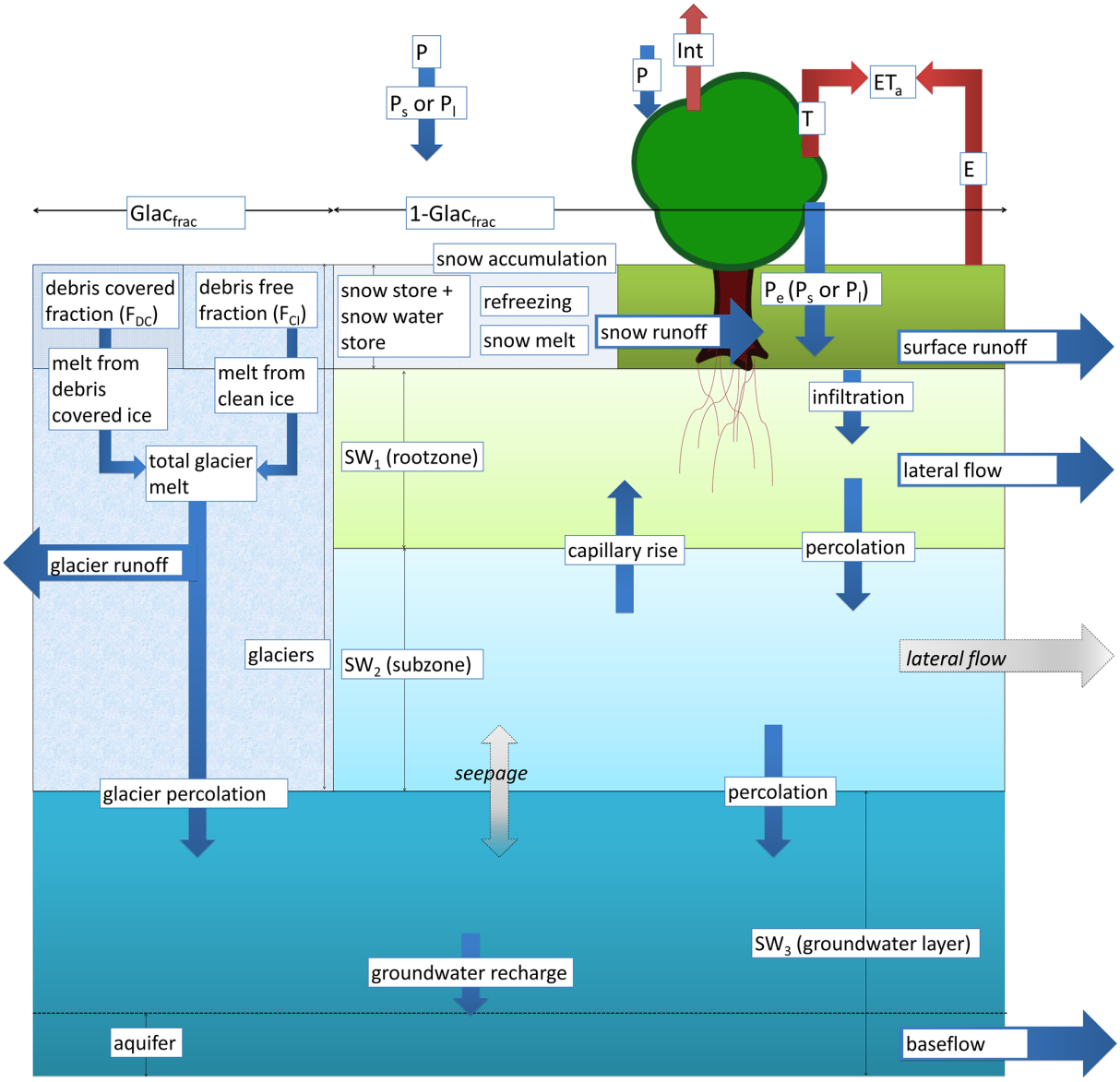


Figure 3.1: SPHY model structure. (copied from Terink et al., 2015).

3.2 Current routing procedure

Total cell-specific runoff available for routing through the stream network consists of different runoff components, namely: rainfall runoff, snow runoff, glacier runoff and baseflow (see Eq. 3.1). Rainfall runoff consists of surface runoff and lateral flow from the first soil layer. Baseflow is resulting from the groundwater module, however when this module is switched off, lateral flow from the second soil layer is considered baseflow.

$$\mathrm{QTot} = \mathrm{RRo} + \mathrm{SRo} + \mathrm{GRo} + \mathrm{BF} \tag{3.1}$$

Where:

QTot	:	Specific runoff on day t (mm)
RRo	:	Rainfall runoff (mm)
SRo	:	Snow runoff (mm)
GRo	:	Glacier runoff (mm)
BF	:	Base Flow (mm)

The SPHY user can select a simple routing scheme or a more complex routing scheme in which discharge from reservoirs/lakes can be included as well through a Q-h relation. From a Digital Elevation Model (DEM) the flow direction network map is derived, which is used in both routing methods.

3.2.1 Simple routing scheme

Most advanced routing methods, e.g. solving the full dynamic Saint Venant equations and Manning equation, require data on channel geometry and morphology which are often not available for the spatial scale that SPHY generally is applied to. Therefore, SPHY uses a simplified routing method in which the accumulated cell-specific runoff flows into its neighbouring cell. This accumulated cell-specific runoff comprises the amount of water in the cell itself plus the amount of water in the cells upstream and is calculated using the PCRaster function *accuflux*. A flow recession coefficient (kx) has been implemented in SPHY to account for flow delay, which is needed to prevent all the specific runoff generated within the catchment on one day to end up at the most downstream part of the catchment on that same day. If preferred the four streamflow contributors can also be routed separately (Terink et al., 2015).

$$\mathrm{QTot}_t^* = \frac{\mathrm{QTot}_t \cdot 0.001 \cdot \mathrm{A}}{24 \cdot 3600} \tag{3.2}$$

$$\mathrm{QTot}_{accu,t} = \mathrm{accuflux}(\mathrm{F}_{dir}, \mathrm{QTot}_t^*) \tag{3.3}$$

$$\mathrm{QTot}_{rout,t} = (1 - \mathrm{kx}) \cdot \mathrm{QTot}_{accu,t} + \mathrm{kx} \cdot \mathrm{QTot}_{rout,t-1} \tag{3.4}$$

Where:

QTot_t^*	:	Total cell-specific runoff on day t ($\mathrm{m}^3\ \mathrm{s}^{-1}$)
QTot_t	:	Total cell-specific runoff on day t (mm)
A	:	Grid-cell area (m^2)
$\mathrm{QTot}_{accu,t}$	:	Accumulated streamflow on day t without flow delay ($\mathrm{m}^3\ \mathrm{s}^{-1}$)
$\mathrm{QTot}_{rout,t}$	:	Routed streamflow on day t ($\mathrm{m}^3\ \mathrm{s}^{-1}$)
$\mathrm{QTot}_{rout,t-1}$	:	Routed streamflow on day *t-1* ($\mathrm{m}^3\ \mathrm{s}^{-1}$)
F_{dir}	:	Flow direction network
kx	:	Flow recession coefficient, ranging from 0 (fast) to 1 (slow) (-)

3.2.2 Advanced routing scheme

The second more advanced routing option is the fractional accumulation flux routing scheme, which is used in the presence of lakes or reservoirs. Lakes/reservoirs act as a buffer from which the water is released delayed. The scheme uses the *accufractionflux* and *accufractionstate* PCRaster functions, which calculate for each cell the fraction of the accumulated water that flows out of the cell and the fraction that is stored in the cell respectively. The fraction that is transported out of the cell equals 1 for non-lake cells. For lake-cells however, the volume that becomes available for routing depends on the actual lake storage and the Q-h relation that is provided. Subsequently, this flux will follow the simple flow accumulation routing scheme. Specifics on data requirements and how to use this routing method are provided in section 2.8.2 of Terink et al. (2015).

$$\mathrm{QTot}_{accu,t} = \mathrm{accufractionflux}(\mathrm{F}_{dir}, \mathrm{S}_{act,t}, \mathrm{Q}_{frac,t}) \tag{3.5}$$

$$\mathrm{S}_{act,t+1} = \mathrm{accufractionstate}(\mathrm{F}_{dir}, \mathrm{S}_{act,t}, \mathrm{Q}_{frac,t}) \tag{3.6}$$

Where:

$\mathrm{S}_{act,t}$: Actual storage (m^3)
$\mathrm{S}_{act,t+1}$: Updated storage to be used in next time step (m^3)

4. Potential routing methods

The dynamics of water in a channel can be described and approximated at different levels of complexity. The full dynamic wave equations, consisting of the continuity equation and the momentum equation, are considered to most accurately describe one-dimensional unsteady flow in open channels (Miller, 1984). However, depending on the application the importance of the various terms in these equations can reduce, which allows us to omit them to simplify the problem. These approximations comprise the continuity equation combined with various simplifications of the momentum equation (Brunner, 1992). This chapter will provide a short description of the full dynamic wave model and of two approximations, i.e. the convection-diffusion and kinematic wave approximation.

4.1 Dynamic wave equations

Dynamic wave routing involve solving the complete one-dimensional Saint Venant flow equations, which consists of the continuity equation (Eq. 4.1) and momentum equation (Eq. 4.2). Assumptions made in deriving these equations are: 1) hydrostatic pressure distribution prevails, hence vertical accelerations are negligible, 2) uniformly distributed velocity across any channel section , 3) small channel bed slope, 4) homogeneous and incompressible flow, and 5) momentum resulting from lateral flow is negligible (Brunner, 1992; Miller, 1984; Litrico and Fromion, 2009; Barati et al., 2012; Shultz, 2007).

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{4.1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial h}{\partial x} - gA(S_0 - S_f) = 0 \tag{4.2}$$

Where:

A	:	Cross-sectional area (m^2)
Q	:	Discharge ($m^3\ s^{-1}$)
x	:	Distance along channel (m)
t	:	Time (s)
g	:	Gravitational acceleration ($m\ s^{-2}$)
h	:	Water depth (m)
S_0	:	Bed slope ($m\ m^{-1}$)
S_f	:	Friction slope ($m\ m^{-1}$)

The terms of the equation of momentum can be described as: 1) local acceleration, 2) convective acceleration, 3) pressure gradient and 4) gravitational and frictional forces (Singh, 1996; Brunner, 1992; Miller, 1984; Shultz, 2007).

The dynamic wave equations are used in various applications, such as river flow forecasting, wave routing in shallow water bodies, dam break flood wave routing, sewer modelling and more in general when a system is subjected to backwater effects due to restrictions downstream such as weirs (Litrico and Fromion,

2009; Brunner, 1992). Since the equations have no general analytical solution it requires numerical techniques, such as the finite element method, finite volume method and finite difference method to solve the non-linear unsteady flow equations (Barati et al., 2012). Models such as HEC-RAS and MIKE 11 solve the full dynamic Saint Venant equations using an implicit finite difference method (Brunner, 2010; MIKE by DHI, 2009). To solve the Saint Venant equations a lot of data is required related to channel geometry and morphology (Barati et al., 2012; Litrico and Fromion, 2009). This data is not always available, which requires simplification of the equations in which some terms of the momentum equation are neglected.

4.2 Convection-diffusion approximation

The convection-diffusion wave (cD-wave) or diffusion wave is based on the continuity equation and a simplification of the momentum equation, in which the acceleration terms have been omitted. This assumption is made since the two acceleration terms counterbalance each other during rising and recession limbs of a flood wave. Furthermore, in most cases the acceleration terms are significantly smaller than the pressure gradient term (Singh, 1996; Shultz, 2007; Torfs, 2002). In most real world cases flood waves are diffusion waves. Equation 4.2 therefore reduces to equation 4.3, in which the pressure term is responsible for describing physical diffusion of the flood wave (Brunner, 1992; Shultz, 2007).

$$\frac{\partial h}{\partial x} - (S_0 - S_f) = 0 \tag{4.3}$$

The continuity equation can be expressed as:

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = 0 \tag{4.4}$$

Where:

u : velocity (m s^{-1})

Combining equations 4.3 and 4.4 the linearized form of the cD-equation can be written as (Lighthill and Whitham, 1955; Torfs, 2002; Singh, 1996; Hayami, 1951):

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} - D\frac{\partial^2 Q}{\partial x^2} = 0 \tag{4.5}$$

Where:

c : wave celerity (m s^{-1})
D : diffusion coefficient (m^2 s^{-1})

The cD-equation is able to describe the translation (c) and the attenuation (D) of a wave as it propagates through the channel (Singh, 1996; Torfs, 2002). However, the cD-equation is not suitable to apply when the cross-sectional geometry of a river changes considerably or when the slope increases largely, as the acceleration terms would not be negligible in those cases. An advantage of this approximation is that it needs less hydro-morphological data and most natural slow to moderately rising flood waves can be described using this approximation (Brunner, 1992). Implicit numerical schemes to solve the cD-equation require boundary conditions both upstream and downstream, implying that the downstream boundary condition should be known or can be estimated (Singh, 1996).

In literature little can be found on typical values for c and D. Vakgroep Hydraulica en Afvoerhydrologie (Year unknown) found values for c ranging from 1.1 – 2.2 m s^{-1} and D values ranging from 10.000 – 40.000 m^2 s^{-1} for different sections of the Rhine near Lobith. Although the conditions of the Rhine river and the Tamakoshi river are far from similar, these values give a sense of the order of magnitude of these parameters.

4.3 Kinematic wave approximation

An even more simplified approximation of the full dynamic wave model is the kinematic wave model, which additionally omits the pressure gradient term. A kinematic wave develops when gravitational and frictional forces approach equilibrium. This is achieved when changes in depth and velocity with respect to distance and change in velocity with respect to time are small compared to channel bed slope. Therefore equation 4.2 reduces further to equation 4.6 (Torfs, 2002; Brunner, 1992; Singh, 1996; Miller, 1984).

$$S_0 = S_f \tag{4.6}$$

The kinematic wave model does not allow for wave attenuation following the assumption that the pressure gradient term is negligible. Therefore the kinematic wave model is most accurate in urban environments where channels are fairly steep and that there is a relationship between discharge, flow depth and position along the channel (Lighthill and Whitham, 1955; Singh, 1996; Shultz, 2007). The linearized form of the kinematic wave equation can be written as:

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = 0 \tag{4.7}$$

This equation describes the translation of a wave through a channel without hydrograph diffusion. However, depending on the numerical scheme used to solve this equation numerical diffusion can be introduced. Furthermore, backwater effects cannot be simulated using the kinematic wave equations because the model allows disturbances only to travel in downstream direction (Singh, 1996; Brunner, 1992; Miller, 1984).

4.4 Selecting a routing method

The appropriate routing method should be selected depending on the characteristics of a hydrological system, the characteristics of the flood wave, data availability, required computational efficiency and the degree of influence of backwater effects.

Comparing the three methods it can be concluded that the cD-wave equations provide a solid approximation of the full dynamic wave equations. It balances between the accuracy and high data demanding dynamic wave model and the simplicity of the kinematic wave model (Shultz, 2007). Because there is no data on channel geometry and discharge data is scarce for the Tamakoshi river the cD-equation is selected as most suitable routing method for the aims of this research project. Further analysis in this report will therefore focus on applying the cD-equation as a routing method.

Additionally, the routing procedure will be executed in a vector layer that is decoupled from the PCRaster environment. This will enable us to execute the dynamic routing procedure on a different temporal and spatial scale than the model simulation that is performed in the raster grid of PCRaster. The aim is therefore also not necessarily to exactly understand and calculate the dynamics of the open water using the full Saint Venant equations, rather than to analyse the pragmatic feasibility of implementing a routing procedure using a vector layer.

5. Numerical approximation

Non-linear partial differential equations have no general analytical solution, which is why numerical discretization techniques are employed to solve them (Miller, 1984). Examples of numerical discretization techniques to solve unsteady flow equations are the finite element method (FEM), finite volume method (FVM) and finite difference method (FDM). The FDM is appropriate when no complex geometries such as uniform grids need to be handled. Mass is more rigorously conserved using a FVM, and its accuracy and computational times is comparable to FDMs. The FEM is more complex to implement and would require more computational effort. When a problem can be solved with a regularly structured grid, FDM can be more efficient to use (Botte et al., 2000; Chung, 2010; Shukla et al., 2011). We consider the assumption of applying a regular structured grid to be appropriate for routing purposes, especially considering the lack of information and data about channel geometry. Therefore we will focus on FDMs.

5.1 Finite-difference schemes

The FDM is based on applying a local Taylor expansion to approximate the derivatives. It uses a regular network of nodes separated by distance dx to discretize a continuous model domain. In a similar way the time domain is discretized using a time step dt (Shultz, 2007; Koohafkan, 2016). There are many choices that can be made selecting an appropriate numerical scheme. Most FDMs will introduce artificial diffusion when applied to the convection part of the cD-equation, which might be larger than the actual physical diffusion (Verma et al., 2012; Chung, 2010). Space derivatives are approximated using central difference, but temporal derivatives can be approximated using three different basic schemes: explicit, implicit and using Crank-Nicolson schemes which combines the two (see Fig. 5.1).

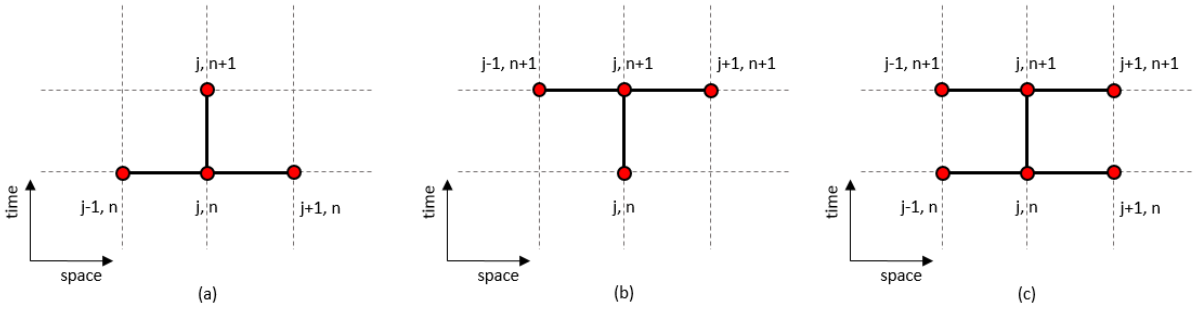


Figure 5.1: (a) Explicit finite-difference discretization, (b) implicit finite-difference discretization, (c) Crank-Nicolson finite-difference discretization.

5.1.1 Explicit finite-difference

A finite-difference scheme is said to be explicit when the value of a variable is computed forward in time explicitly depending on the value of the previous time step (see Fig. 5.1a). An explicit method is advantageous over an implicit method considering the ease of programming and the requirements for computer resources as it does not use iterative computations, however a disadvantage is that this method is subject to numerical stability constraints, and should therefore meet the requirements defined by the Courant condition (Equation 5.1) (Koohafkan, 2016). For numerically stable solutions C should

be smaller than 1. Therefore the size of the temporal resolution dt is restricted, assuming a predefined spatial resolution dx and a known wave celerity c.

$$\mathrm{C} = \frac{\Delta t}{\Delta x} c \tag{5.1}$$

Where:

C	:	Courant number (-)
Δt	:	Time step (s)
Δx	:	Spatial step (m)
c	:	Wave celerity (m s^{-1})

The MacCormack scheme is an example of a 2-step explicit FDM which uses a backwards-looking predictor step, followed by a forward-looking corrector step. This order can also be reversed for each time step. This scheme is easier to apply and it provides more accurate results at coarser spatial and temporal resolutions compared to the Lax diffusive scheme. It has been widely used to solve non-linear equations (Hoffman and Frankel, 2001; Pletcher et al., 2012). The MacCormack scheme is one of the options provided in the *rivr*-package for the programming language R to solve the full Saint Venant equations (Koohafkan, 2016).

5.1.2 Implicit finite-difference

An implicit finite-difference scheme numerically solves the equations iteratively as the value for the next time step depends on itself (see Fig. 5.1b). It is unconditionally numerically stable with no restrictions on the size of dt and dx (Barati et al., 2012). However, to achieve high accuracy for a solution, dt should approach the time step determined from the Courant condition.

5.1.3 Crank-Nicolson

A third basic finite-difference scheme is the Crank-Nicolson scheme (see Fig. 5.1c). Suppose a one-dimensional partial differential equation of the form:

$$\frac{\partial u}{\partial t} = f\left(u, x, t, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right) \tag{5.2}$$

The Crank-Nicolson method combines the fully explicit (forward Euler) and fully implicit (backward Euler) schemes by computing the average of those:

$$\frac{u_x^{t+1} - u_x^t}{\Delta t} = f_x^t\left(u, x, t, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(Explicit)} \tag{5.3}$$

$$\frac{u_x^{t+1} - u_x^t}{\Delta t} = f_x^{t+1}\left(u, x, t, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right) \qquad \text{(Implicit)} \tag{5.4}$$

$$\frac{u_x^{t+1} - u_x^t}{\Delta t} = \frac{1}{2}\left[f_x^{t+1}\left(u, x, t, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right) + f_x^t\left(u, x, t, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}\right)\right] \qquad \text{(Crank-Nicolson)} \tag{5.5}$$

Since it is an implicit method because the solution does not explicitly depend on the value of the previous time step, it is numerically stable. However, it is prone to oscillations if the time step relative to the spatial step becomes large. It is a second order method because it is centered in time. For one-dimensional (convection-) diffusion problems this method is often resorted to and it generally yields accurate results (Crank and Nicolson, 1947; Smith, 1985).

5.2 Boundary conditions

To find a numerical solution for a differential equation, initial conditions and boundary conditions need to be specified. There are three commonly used types of boundary conditions. A Dirichlet boundary condition specifies that the state of the dependent variable at the boundary is a known function of time, i.e. in hydraulics this can be defined as fixed head. This boundary condition is relatively easy to implement (Singh, 1996; Shahedi, 2008). Neumann boundary condition constitute the case in which the derivative of the dependent variable is specified, i.e. in hydraulics a fixed flux. When a head-dependent flux is specified at the boundary this is called a mixed boundary condition or Robins boundary condition. It is a linear combination of the Dirichlet and Neumann boundary conditions and specifies a known total flux over the boundary which comprises a diffusion and convection component.

Upstream typically a stage or flow hydrograph is provided, while downstream also a rating curve (Q-h relation) or normal depth (the depth of flow when the slope of the water surface equals the slope of the channel bottom (Singh, 1996)) can be specified to define the boundary condition (Brunner, 2010). The downstream boundary should be chosen such that the backwater effect is minimal. Assuming that backwater effect can be neglected in case of a situation like Tamakoshi river is valid because of the topographical height differences. For a case in The Netherlands this assumption might not hold because of artificial constructions that regulate water levels. Minimizing the influence of backwater can be done in different ways for example: 1) let the river widen dramatically at the downstream point, i.e. artificially creating a situation that simulates conditions in which all water drains into a sea, 2) artificially extend the length of the river such that the possible backwater effect caused by errors in boundary specification will have minimal effect on the actual end of the river, or 3) Provide a Q-h relation when data is available.

The SPHY model simulates the specific runoff for each cell in the raster. This specific runoff consists of surface runoff, lateral flow, baseflow, snowmelt and glacier melt. From this the accumulated runoff for the upstream location of every reach of the steam network has been calculated. This hydrograph is used as the upstream boundary condition. Since there is no data available from which a Q-h relation could be determined the same method has been used for the outlet of the entire stream network. The downstream hydrographs for the smaller individual reaches are derived from results obtained by applying the cD-equation from down- to upstream as a preprocessing step (for further explanation see section 6.8)

5.3 Numerical discretization of cD-equation

The convection-diffusion equation is discretized using the Crank-Nicolson method. Consider the cD-equation:

$$\frac{\partial Q}{\partial t} = D\frac{\partial^2 Q}{\partial x^2} - c\frac{\partial Q}{\partial x} + f(Q) \tag{5.6}$$

Where:

f : Reaction term, e.g. lateral flow

Applying the Crank-Nicolson method to the cD-equation, and adding the option to shift between predominantly implicit or explicit (i.e. $\omega = 1 \rightarrow$ fully implicit, $\omega = 0 \rightarrow$ fully explicit) yields:

$$\begin{aligned}\frac{Q_x^{t+1} - Q_x^t}{\Delta t} = \frac{D}{2\Delta x^2}&\left((1-\omega)\left(Q_{x+1}^t - 2Q_x^t + Q_{x-1}^t\right) + \omega\left(Q_{x+1}^{t+1} - 2Q_x^{t+1} + Q_{x+1}^{t+1}\right)\right) \\ &- \frac{c}{4\Delta x}\left((1-\omega)\left(Q_{x+1}^t - Q_{x-1}^t\right) + \omega\left(Q_{x+1}^{t+1} - Q_{x-1}^{t+1}\right)\right) + f\left(Q_x^t\right)\end{aligned} \tag{5.7}$$

Multiplying with Δt and defining $\sigma = \frac{\mathrm{D}\Delta\mathrm{t}}{2\Delta\mathrm{x}^2}$ and $\rho = -\frac{\mathrm{c}\Delta\mathrm{t}}{4\Delta\mathrm{x}}$ yields:

$$\begin{aligned}\omega\left(Q_x^{t+1} + 2\sigma Q_x^{t+1} - \sigma Q_{x-1}^{t+1} + \rho Q_{x-1}^{t+1} - \sigma Q_{x+1}^{t+1} - \rho Q_{x+1}^{t+1}\right) = \\ (1-\omega)\left(Q_x^t - 2\sigma Q_x^t + \sigma Q_{x-1}^t - \rho Q_{x-1}^t + \sigma Q_{x+1}^t + \rho Q_{x+1}^t\right) + \Delta t f\left(Q_x^t\right)\end{aligned} \tag{5.8}$$

After reordering the equation becomes:

$$\begin{aligned}(-\sigma+\rho)\omega Q_{x-1}^{t+1} + (1+2\sigma\omega)Q_x^{t+1} - (\sigma+\rho)\omega Q_{x+1}^{t+1} = \\ (\sigma-\rho)(1-\omega)Q_{x-1}^t + (1-2\sigma(1-\omega))Q_x^t + (\sigma+\rho)(1-\omega)Q_{x+1}^t + \Delta t f\left(Q_x^t\right)\end{aligned} \tag{5.9}$$

This equation is valid for spatial indices x=1, ..., X-2, in which X is the number of discrete spatial points in the grid. However, for the points at the boundaries, i.e. x=0 and x=X-1 the equation makes no sense because Q_{-1}^t and Q_X^t are located outside the grid:

$$\begin{aligned}\mathbf{x = 0}: (-\sigma+\rho)\omega Q_{-1}^{t+1} + (1+2\sigma\omega)Q_0^{t+1} - (\sigma+\rho)\omega Q_1^{t+1} = \\ (\sigma-\rho)(1-\omega)Q_{-1}^t + (1-2\sigma(1-\omega))Q_0^t + (\sigma+\rho)(1-\omega)Q_1^t + \Delta t f\left(Q_0^t\right)\end{aligned} \tag{5.10}$$

$$\begin{aligned}\mathbf{x = X\text{-}1}: (-\sigma+\rho)\omega Q_{X-2}^{t+1} + (1+2\sigma\omega)Q_{X-1}^{t+1} - (\sigma+\rho)\omega Q_X^{t+1} = \\ (\sigma-\rho)(1-\omega)Q_{X-2}^t + (1-2\sigma(1-\omega))Q_{X-1}^t + (\sigma+\rho)(1-\omega)Q_X^t + \Delta t f\left(Q_{X-1}^t\right)\end{aligned} \tag{5.11}$$

However, by providing at both the upstream and downstream boundaries a specified flow hydrograph, which can be interpreted as a Dirichlet boundary as a function of time, the numerical approximation can be written compactly in a linear system using vector notation that looks like:

A * Q[t+1] = B * Q[t] + f[t]

in which:

$$\mathrm{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ (-\sigma+\rho)\omega & 1+2\sigma\omega & (-(\sigma+\rho))\omega & 0 & \cdots & 0 \\ 0 & (-\sigma+\rho)\omega & 1+2\sigma\omega & (-(\sigma+\rho))\omega & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & (-\sigma+\rho)\omega & 1+2\sigma\omega & (-(\sigma+\rho))\omega & 0 \\ 0 & \cdots & 0 & (-\sigma+\rho)\omega & 1+2\sigma\omega & (-(\sigma+\rho))\omega \\ 0 & \cdots & 0 & 0 & 0 & 1 \end{bmatrix}, \; \mathrm{Q}^{t+1} = \begin{bmatrix} Q_1^{t+1} \\ Q_2^{t+1} \\ Q_3^{t+1} \\ \vdots \\ Q_{X-2}^{t+1} \\ Q_{X-1}^{t+1} \\ Q_X^{t+1} \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ (\sigma-\rho)(1-\omega) & 1-2\sigma(1-\omega) & (\sigma+\rho)(1-\omega) & 0 & \cdots & 0 \\ 0 & (\sigma-\rho)(1-\omega) & 1-2\sigma(1-\omega) & (\sigma+\rho)(1-\omega) & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & (\sigma-\rho)(1-\omega) & 1-2\sigma(1-\omega) & (\sigma+\rho)(1-\omega) & 0 \\ 0 & \cdots & 0 & (\sigma-\rho)(1-\omega) & 1-2\sigma(1-\omega) & (\sigma+\rho)(1-\omega) \\ 0 & \cdots & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathrm{Q}^t = \begin{bmatrix} ,Q_1^t \\ Q_2^t \\ Q_3^t \\ \vdots \\ Q_{X-2}^t \\ Q_{X-1}^t \\ Q_X^t \end{bmatrix}, \; \mathrm{f}^t = \begin{bmatrix} \Delta t f(Q_1^t) \\ \Delta t f(Q_2^t) \\ \Delta t f(Q_3^t) \\ \vdots \\ \Delta t f(Q_{J-2}^t) \\ \Delta t f(Q_{J-1}^t) \\ \Delta t f(Q_J^t) \end{bmatrix}$$

When enough data is available a Q-h relation could be implemented at the downstream boundary. A Q-h relation can be described by a linear combination of the Dirichlet and Neumann boundary condition:

$$\alpha * \text{Neumann} + \beta * \text{Dirichlet} = \gamma \tag{5.12}$$

$$\alpha \cdot \frac{\partial \text{Q}}{\partial \text{x}}(\text{X-1,t}) + \beta \cdot \text{Q(X-1,t)} = \gamma \tag{5.13}$$

Discretization of equation 5.13 using centered difference for x = X-1 (i.e. boundary node), gives:

$$\alpha \cdot \frac{\text{Q}^t_X - \text{Q}^t_{X-2}}{2\Delta x} + \beta \cdot \text{Q}^t_{X-1} = \gamma \tag{5.14}$$

$$-\text{Q}^t_{X-2} + \frac{2\beta\Delta\text{x}}{\alpha}\text{Q}^t_{X-1} + \text{Q}^t_X = \frac{2\Delta\text{x}\gamma}{\alpha} \tag{5.15}$$

Following equation 5.15 the last three terms of the last row in matrix A and matrix B should be changed to -1, $\frac{2\beta\Delta\text{x}}{\alpha}$, and 1 from left to right respectively and the last term of vector f should become $\frac{2\Delta\text{x}\gamma}{\alpha}$ for a Robin boundary condition. Due to the low data availability a Q-h relation could not be determined for the Tamakoshi river basin. Therefore we are restricted to use the flow hydrograph that could be constructed from the accumulated discharge produced in each cell as simulated by SPHY. However, to make the solution numerically more stable, the linear combination of the Dirichlet and Neumann boundary condition as described in equation 5.15 can be used. For this β will be chosen 1.0 and γ will be the 'observed' flow hydrograph, which in this case is the accumulated discharge simulated by SPHY. By also giving some weight to α (e.g. $\alpha = 0.2$), and thus including the gradient of discharge (i.e. Neumann boundary condition), the solution will numerically become more stable because the gradient of the discharge will not take unrealistic values at the boundary.

6. Advanced routing module implementation

The advanced routing procedure comprises the use of the convection-diffusion equation (cD-equation) for channel flow routing through a drainage network. This method will allow to simulate the shift and attenuation of a wave as it propagates through the stream network. The cD-equation needs to be applied to every reach of the stream network using appropriate initial and boundary conditions. The simulated hydrograph at the outlet of each reach will act as lateral inflow into the connected next reach of a higher stream order. The implementation of this procedure will be described in this chapter. All programming code in this chapter is included in the python script '*CDrouting.py*', unless stated otherwise.

6.1 Preprocessing

6.1.1 Preparing data and maps

Several preparatory steps needs to be performed before the routing procedure can be started. The programming code for this is provided below (r.27–73). The command lines within the code explain which step is performed.

```
27 # Read the input and output directories from the configuration file
28 inpath = config.get('DIRS', 'inputdir')
29 outpath = config.get('DIRS', 'outputdir')

31 # Missing value definition
32 MV= -9999

34 # Set the timing criteria
35 import datetime
36 datetime = datetime

38 sy = config.getint('TIMING', 'startyear')
39 sm = config.getint('TIMING', 'startmonth')
40 sd = config.getint('TIMING', 'startday')
41 ey = config.getint('TIMING', 'endyear')
42 em = config.getint('TIMING', 'endmonth')
43 ed = config.getint('TIMING', 'endday')
44 startdate = datetime.datetime(sy,sm,sd)
45 enddate = datetime.datetime(ey,em,ed)

47 # Visualisation of results in maps
48 visFLAG = config.getint('CDROUTING','VisualisationFLAG')

50 # Set clonemap
51 clonemap = inpath + config.get('GENERAL','mask')
52 pcr.setclone(clonemap)
```

```
53 clone = pcr.ifthen(pcr.readmap(clonemap), pcr.boolean(1))
54 pcr.report(clone, outpath+'clone.map')

56 # Adjusted DEM based on the created ldd map
57 DEMadj = pcr.lddcreatedem(pcr.readmap(inpath +
   config.get('GENERAL', 'dem')), 1e31, 1e31, 1e31, 1e31)
58 pcr.report(DEMadj, outpath+'demadj.map')
59 DEM = pcr.readmap(outpath + 'demadj.map')

61 # Load locations map where time-series will be recorded and determine row and
   # column of the locations.
62 stations = pcr.readmap(inpath + config.get('GENERAL','locations'))
63 stations2 = pcr.ifthenelse(stations!=MV, pcr.boolean(1), pcr.boolean(0))
64 stations3 = pcr.ifthen(stations2,DEM)
65 stations_loc = numpy.argwhere(pcr.pcr2numpy(stations3, MV)!=MV)
66 stations_loc = numpy.hstack((stations_loc, numpy.zeros((stations_loc.shape[0], 3))))
67 # Read station names
68 names_table = inpath + config.get('GENERAL','names')
69 stations_name = pcr.lookupscalar(names_table, stations)
70 names = pcr.pcr2numpy(stations_name, MV)

72 # Read and set runoff forcing
73 Runoff = outpath + config.get('CDROUTING','totrun')
```

6.1.2 Define the stream network

A map containing the local drain direction is calculated based on the Digital Elevation Model (DEM) using PCRaster in order to define the stream network of the basin (r.75–76). The PCRaster map of the local drain direction is converted into a numpy array (r.81). In this way the values can be adjusted such that pits are created at the inlets of the reaches of the smallest streamorder, and additionally at the outlets of all reaches in the stream network. This will be performed at a later stage (see Sect. 6.1.4). Every cell in the raster will be assigned a flow direction, and therefore every cell is considered to be part of the stream network. To reduce the refinement of the network and to create a network of well-defined streams in which not all cells are assigned to the stream network, a threshold needs to be specified in the SPHY config-file (r.83–85). This threshold specifies the minimal area upstream of a cell (in km^2) required for this cell to be assigned to the stream network. Using the spatial resolution of the raster (r.138–141) this area is converted into number of cells using equation 6.1 (r.91). To determine the number of cells upstream of a cell the 'accuflux' function as incorporated in PCRaster has been used (r.89–90). This function calculates the accumulated amount of material flowing out of a cell, which consists of the accumulated material produced upstream of the cell added to the material in the cell itself (Karssenberg et al., 2001). Using the number '1' as material, will provide the number of cells upstream of a cell (including the cell itself). If this number is larger than the threshold value, the cell will be assigned to the stream network (r.92–93).

$$\text{Threshold}_{\text{Number of cells}} = \frac{\text{Threshold}_{\text{Area}}}{(\text{Spatial resolution})^2} \tag{6.1}$$

Where:

$\text{Threshold}_{\text{Number of cells}}$	:	Threshold defining number of cells upstream of cell for this cell to be assigned to stream network (-)
$\text{Threshold}_{\text{Area}}$	:	Minimal area required upstream of a cell for this cell to be assigned to stream network (km^2)
Spatial resolution	:	Spatial resolution of the raster (km)

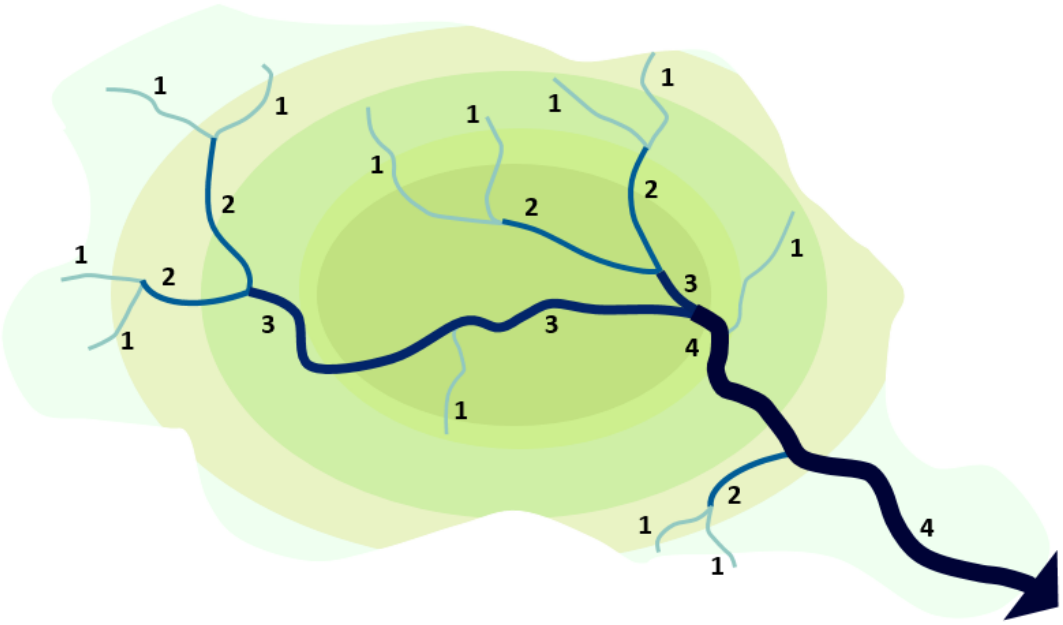
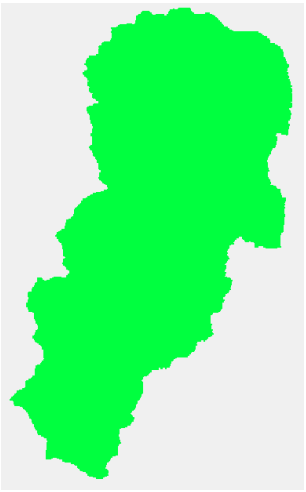
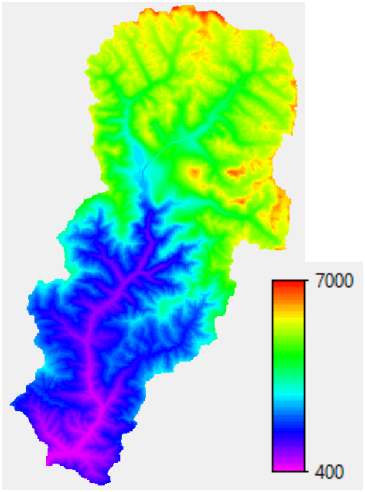


Figure 6.1: River network demonstrating Strahlers principle to determine the stream order.

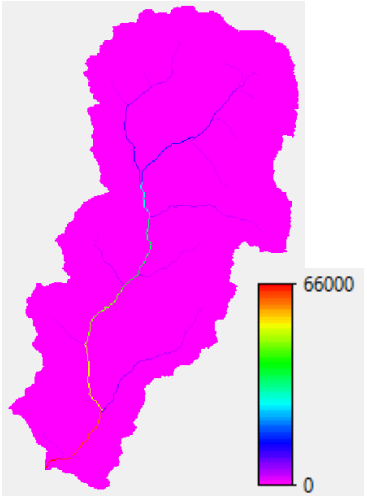
```
75  # Calculate local drain direction using Digital Elevation Model (DEM)
76  FlowDir_ini = pcr.lddcreate(DEM, 1e31, 1e31, 1e31, 1e31)
77  # Convert the PCRmap into numpy array 'ldd_inout'.
78  # This array will be used to create pits at the inlets of the reaches of
79  # the smallest streamorder and additionally pits at all outlets.
80  # This is needed to substract the water from the system at these points.
81  ldd_inout = pcr.pcr2numpy(FlowDir_ini, MV)

83  # Threshold specifies the minimum area (in km^2) required upstream of a cell,
84  # for this cell to be assigned to the stream network.
85  threshold_area = config.getfloat('CDROUTING', 'threshold')

#### FROM SCRIPT 'sphy.py' ####
138  # Spatial resolution of PCRaster grid
139  self.SpaceRes = pcr.cellarea()
140  self.SpaceRes = pcr.pcr2numpy(self.SpaceRes,self.MV)
141  self.SpaceRes = self.SpaceRes[0][0]**0.5/1000
####

87  # Determine stream network (if more than #...cells upstream, then it is considered)
88  # a stream).  Assign '1' to streams and '0' to non-streams.
89  accuflux = pcr.accuflux(FlowDir_ini,1)
90  pcr.report(accuflux, outpath + 'accuflux.map')
91  threshold_cells = int(threshold_area/((SpaceRes/1000)**2))
92  network = pcr.ifthenelse(accuflux > threshold_cells, pcr.nominal(1), pcr.nominal(0)
93  pcr.report(network, outpath + 'network.map')
```



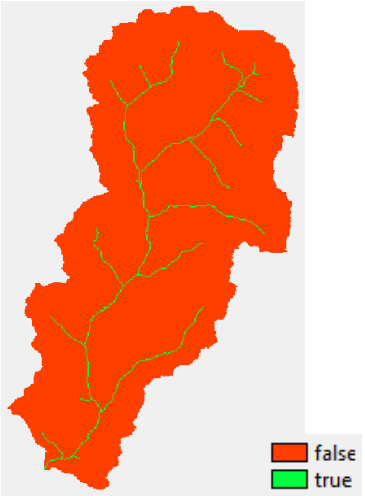
(a) Clone map indicating the area of interest (r.48)



(b) Digital Elevation Model (DEM) (r.52)



(c) Accuflux.map (r.82)



(d) Stream network.map based on threshold value (r.85)

Figure 6.2: Define the stream network using the DEM.

6.1.3 Determine stream order

Once the stream network has been defined the stream order of all reaches belonging to the stream network are specified using the function 'streamorder' as implemented in PCRaster (r.99–101). This function again requires the local drain direction map as input, and therefore will use the map in which the ldd has been determined for all cells belonging to the stream network (r.95–97). To obtain the stream order solely for the defined stream network, the two maps are multiplied (r.103–105). The minimum and maximum stream order are retrieved from the resulting map (r.107–110).

```
95  # Determine flow direction only for stream network
96  FlowDir_network = pcr.ldd(pcr.scalar(network)*pcr.scalar(FlowDir_ini))
97  pcr.report(FlowDir_network, outpath + 'FlowDir_network.map')

99  # Determine the streamorder map from the FlowDir map
100  streamorder = pcr.streamorder(FlowDir_ini)
101  pcr.report(streamorder, outpath+'streamorder.map')
```

```
103 # Determine for the defined network what the streamorder is
104 orderNetwork = pcr.nominal(pcr.scalar(streamorder)*pcr.scalar(network))
105 pcr.report(orderNetwork, outpath + 'orderNetwork.map')

107 # Determine the minimum and maximum streamorder.  For this convert to numpy array,
    # make all missing values 0 and flatten it (hstack)
108 orders = numpy.hstack(pcr.pcr2numpy(orderNetwork, 0))
109 strordmin = int(min(i for i in orders if i > 0))
110 strordmax = int(max(i for i in orders if i > 0))
```

6.1.4 Identify reaches

From the '*orderNetwork*' map all reaches per stream order are retrieved and saved in a separate map, containing all reaches for the regarded stream order (r.119 & 122). Additionally, the location (x,y) of all cells belonging to that stream order are stored in table *reachID* (r.126–128).

In general when performing a function using PCRaster, it will scan the raster row by row. This therefore determines the order in which cells are stored when they meet a certain criteria. Since this order does not necessarily coincide with downstream neighbouring cells another method was founded to determine neighbouring cells. For this all cells belonging to a reach the value of the DEM (r.120 & 123) and Slopelength are calculated (r.121 & 124). At first it was expected that DEM values would systematically decrease in downstream direction, however due to some small inconsistencies in the map this is not always the case. Slopelength is a function in PCRaster that calculates the accumulative-friction-distance of the longest path upstream (r.112–115), which from a pragmatic point of view, means that this value systematically increases in downstream direction.

```
112 # Calculate slopelength - calculates the accumulative friction-distance of the
113 # longest path upstream.Needed to sort the cells of a reach in downstream direction.
114 slopelength = pcr.slopelength(FlowDir_network,1)
115 pcr.report(slopelength, outpath+'slopelength.map')

117 # Determine the reaches and its indices on the map,
    # as well as for these reaches subtract DEM and SlopeLength values.
118 for order in range(strordmin,strordmax+1):
119     reach = pcr.ifthenelse(orderNetwork==order, pcr.boolean(1), pcr.boolean(0))
120     demreach = pcr.ifthen(reach,DEM)# if part of the reach, then give DEM value
121     SLreach = pcr.ifthen(reach,slopelength)
122     pcr.report(reach, outpath + 'reach_' + str(order) + '.map')
123     pcr.report(demreach, outpath + 'demreach_' + str(order) + '.map')
124     pcr.report(SLreach, outpath + 'SLreach_' + str(order) + '.map')

126     # Convert PCRmap to numpy array and determine for the reach the location
        # (row and col), and store this.
127     reachID = numpy.argwhere(pcr.pcr2numpy(SLreach, MV)!=MV)
128     numpy.savetxt(outpath+'reachid_'+str(order)+'.txt', reachID, delimiter=',')
```

The next step is to separate the individual reaches per stream order. First a matrix consisting of 5 columns is created 'ArrayID' (r.163–164). The first column is reserved to assign an ID number to each cell. The second and third column contain the row and the column of the cells of all reaches respectively, indicating their location in the raster (r.167–168). The fourth and fifth column are filled with the DEM and Slopelength values respectively (r.153–161 + 169–170). Subsequently the matrix is sorted from low to high based on Slopelength values (r.172–173), which means that the cells are now ordered in downstream direction. The table is saved as 'ReachArrayID_*order*' (r.177). Now everything is prepared to retrieve the individual reaches per stream order (r.179–260).

Table 6.1: Example of the resulting 'reachID' table which stores the locations of the cells belonging to the specified stream order (r.126–128).

x	y
350	156
360	157
370	157
⋮	⋮
420	650
421	660

For this purpose the outlet of one reach is found and reported (r.186–187) by determining the maximum of the Slopelength map (i.e. the maximum slopelength will be found at the most downstream point of a reach), which is then assigned TRUE (r.182–183). In case there are more than one cells with the same maximum Slopelength value, then also the DEM value will be considered in order to find a just one unique outlet (r.184–185). For this outlet its catchment is determined using the function 'catchment' in PCRaster (r.189–191). This is a sub-catchment of the total considered area. By combining the map of the sub-catchment and the map containing all the reaches of the stream order (r.192–195), the cells of the reach that are located within the sub-catchment are retrieved (r.196) and its locations are saved in a matrix, which again is ordered in downstream direction using Slopelength values and saved in a table named 'SubReachArrayID' (r.198–213). Subsequently these cells are removed from the matrix (r.228–232) that contains all reaches of the regarded stream order so a new maximum can be found and therefore the outlet of the next sub-reach, repeating this procedure until all sub-reaches are identified.

In between it is determined in which specific reach the observation stations are located (r.215–226). At a later stage this allows us to retrieve time series of the routing results at these locations in order to compare these simulations to observations.

Furthermore, the array of the local drain direction map *ldd_inout* is changed into the number '5' for every outlet and for all inlets of reaches with the smallest stream order, as this creates a pit (r.237 & 247). A pit in the local drain direction map will prevent water to flow past this point (see next Section (Sect. 6.2)). The location of the outlets, together with the stream order and sub-reach number are stored in the array 'outIDreach' (r.238–241). The next section will discuss on the need to create these pits. Furthermore a map is created in which only the outlets are given *ldd_out* (r.133 & 236), and similarly a map containing only the inlets of reaches of the smallest stream order *ldd_in* (r.132 & 246). These maps are needed to inject the water, either retrieved from the system or through the open channel routing, to the right locations. In addition a map is created containing only the inlets of reaches with a stream order > 1 *ldd_ups* (r.134 & 250). The latter is used to determine the upstream boundary of each reach when running the cD-equation backwards which is needed to determine the downstream boundary conditions for each reach (see Sect. 6.8).

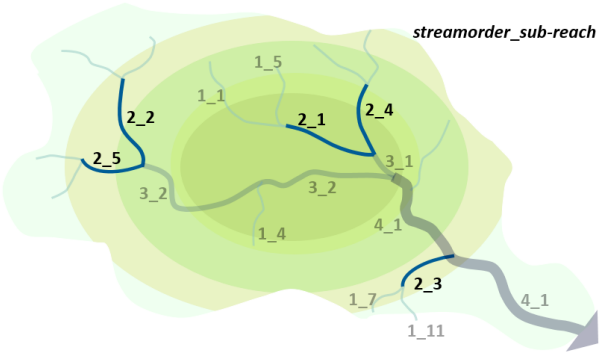


Figure 6.3: River network where all sub-reaches of stream order 2 are highlighted and numbered.

```
130 # Preparation for inlet map (only inlets of smallest streamorder), outlet map and
131 # a map with inlets of all reaches with streamorder>1
132 ldd_in = pcr.pcr2numpy(pcr.ifthen(clone==1, pcr.boolean(0)), MV)
133 ldd_out = pcr.pcr2numpy(pcr.ifthen(clone==1, pcr.boolean(0)), MV)
134 ldd_ups = pcr.pcr2numpy(pcr.ifthen(clone==1, pcr.boolean(0)), MV)

136 # Create empty arrays to store the ID's, 'order' and 'sub' of the outlets,
137 # inlets and inlets of all reaches with streamorder>1 respectively.
138 outIDreach = []
139 inIDreach = []
140 upsIDreach = []

142 ##########
143 # For each stream order order the cells in downstream direction.
144 # An array is produced which contains:  1)Unique ID, 2)row, 3)col, 4)Elevation,
145 # 5)Slopelength (this is used to sort the cells in downstream direction)
146 ##########
147 for order in range(strordmin,strordmax+1):
148     reach = pcr.readmap(outpath + 'reach_' + str(order) + '.map')

150     # Read file containing the indices of the reach
151     reachID = numpy.loadtxt(outpath+'reachid_'+str(order)+'.txt', delimiter=',')

153     # Determine DEM value for each cell of the reach and convert to array
154     dem = pcr.readmap(outpath + 'demreach_' + str(order) + '.map')
155     runoffDEM = pcr.ifthen(reach, dem)
156     DEMArray = pcr.pcr2numpy(runoffDEM,MV)

158     # Determine SlopeLength value for each cell of the reach and convert to array
159     SL = pcr.readmap(outpath + 'SLreach_' + str(order) + '.map')
160     runoffSL = pcr.ifthen(reach, SL)
161     SLArray = pcr.pcr2numpy(runoffSL,MV)

163     # Create empty array with 5 columns and length equal to number of cells
        # in the reach.
164     ArrayID = numpy.zeros((len(reachID),5))
165     # Fill the array:
166     for cell in range(0,len(reachID)):
167         ArrayID[cell,1]=int(reachID[cell,0])
168         ArrayID[cell,2]=int(reachID[cell,1])
169         ArrayID[cell,3]=DEMArray[int(reachID[cell,0]),int(reachID[cell,1])]
170         ArrayID[cell,4]=SLArray[int(reachID[cell,0]),int(reachID[cell,1])]

172     # Sort the array according to SlopeLength
173     ArrIDSorted=ArrayID[numpy.argsort(ArrayID[:,4])]
174     # After sorting add ID value to the cells.  Now the cells are ordered in
        # a downstream way.
175     for ID in range(0,len(reachID)):
176         ArrIDSorted[ID,0]=ID+1
177     numpy.savetxt(outpath + 'ReachArrayID_' + str(order) + '.txt',
        ArrIDSorted, delimiter=',')

179     # Identify and subtract individual sub-reaches within each stream order
180     sub=0
181     while max(ArrIDSorted[:,4])!=MV:
182         # Find the outlet of one of the sub-reaches and make it 'TRUE'
```

```
183         SLoutlet = pcr.boolean(SL == max(ArrIDSorted[:,4]))
184         # Needed in case there are more points with the same maximum value
185         DEMoutlet = pcr.ifthen(SLoutlet,dem)
186         outlet = pcr.boolean(DEMoutlet == (ArrIDSorted[ArrIDSorted[:,4]==
            max(ArrIDSorted[:,4])][0,3]))
187         pcr.report(outlet, outpath + 'outlet' + str(order) + '_' +
            + str(sub+1) + '.map')

189         # Determine for point '1' its catchment
190         subcatch = pcr.catchment(FlowDir_ini,outlet)
191         pcr.report(subcatch, outpath + 'subcatch' + str(order) + '_' +
            str(sub+1) + '.map')
192         # Select from the reaches within this stream order the cells that lie within
            # the defined catchment of the accompanying outlet point
193         subreach=pcr.ifthen(subcatch,reach)
194         subreach2 = pcr.ifthen(subreach,SL)
195         pcr.report(subreach2, outpath + 'subreach' + str(order) + '_' +
            str(sub+1) + '.map')
196         subreachID = numpy.argwhere(pcr.pcr2numpy(subreach2, MV)!=MV)

198         # Create empty array with 6 columns and length equal to number of cells
            # in the reach.
199         subarrayID = numpy.zeros((len(subreachID),6))
200         # Fill the array:
201         for cell in range(0,len(subreachID)):
202             subarrayID[cell,1]=int(subreachID[cell,0])
203             subarrayID[cell,2]=int(subreachID[cell,1])
204             subarrayID[cell,3]=SLArray[int(subreachID[cell,0]),int(subreachID[cell,1])]
205             subarrayID[cell,4]=int(order)
206             subarrayID[cell,5]=int(sub+1)

208         # Sort the array according to SlopeLength
209         subarrIDSorted=subarrayID[numpy.argsort(subarrayID[:,3])]
210         # After sorting add ID value to the cells.
            # Now the cells are ordered in a downstream way.
211         for ID in range(0,len(subreachID)):
212             subarrIDSorted[ID,0]=ID+1
213         numpy.savetxt(outpath + 'SubReachArrayID_' + str(order) + '_' +
            str(sub+1) + '.txt', subarrIDSorted, delimiter=',')

215         # Find in which reach (i.e.  order and sub) the stations are located
216         # and store this in 'stations_loc'.
217         for loc in range(len(stations_loc)):
218             # Exclude streams comprising 1 cell
219             # (assuming that this will not be the location of a station)
220             if numpy.size(subarrIDSorted)>6:
221                 if stations_loc[loc,0] in subarrIDSorted[:,1]:
222                     index = numpy.where(subarrIDSorted[:,1]==stations_loc[loc,0])
223                     if subarrIDSorted[index[0][0],2]==stations_loc[loc,1]:
224                         stations_loc[loc,2]=order
225                         stations_loc[loc,3]=sub+1
226                         stations_loc[loc,4]=index[0][0]

228         # Remove (=set to MV) the cells of the subreach regarded from
            # the array 'ArrIDSorted' in order to find the next max SL
229         for subID in range(0,len(subreachID)):
```

```
230             x=ArrIDSorted[ArrIDSorted[:,1]==subreachID[subID,0]]
231             y=x[x[:,2]==subreachID[subID,1]]
232             ArrIDSorted[int(y[0,0])-1,:]=MV

234         # Create a pit (cell value is 5) in the ldd_inout at the outlet points of
235         # each subreach.  Outlets are the last value of the 'sub array sorted'.
236         ldd_out[int(subarrIDSorted[-1][1]),int(subarrIDSorted[-1][2])]=1
237         ldd_inout[int(subarrIDSorted[-1][1]),int(subarrIDSorted[-1][2])]=5
238         outIDreach.append(int(order))
239         outIDreach.append(int(sub+1))
240         outIDreach.append(int(subarrIDSorted[-1][1]))
241         outIDreach.append(int(subarrIDSorted[-1][2]))
242         # Find index of the most upstream cell of each subreach
243         # and change this in the map to value '1'
244         # Make inlets also pits in ldd_inout array.
245         if order==strordmin:
246             ldd_in[int(subarrIDSorted[0][1]),int(subarrIDSorted[0][2])]=1
247             ldd_inout[int(subarrIDSorted[0][1]),int(subarrIDSorted[0][2])]=5
248         # Change the location of the inlets of reaches with streamorder>1 to '1'
249         if order>strordmin:
250             ldd_ups[int(subarrIDSorted[0][1]),int(subarrIDSorted[0][2])]=1
251             upsIDreach.append(int(order))
252             upsIDreach.append(int(sub+1))
253             upsIDreach.append(int(subarrIDSorted[0][1]))
254             upsIDreach.append(int(subarrIDSorted[0][2]))
255         inIDreach.append(int(order))
256         inIDreach.append(int(sub+1))
257         inIDreach.append(int(subarrIDSorted[0][1]))
258         inIDreach.append(int(subarrIDSorted[0][2]))

260         sub+=1
```

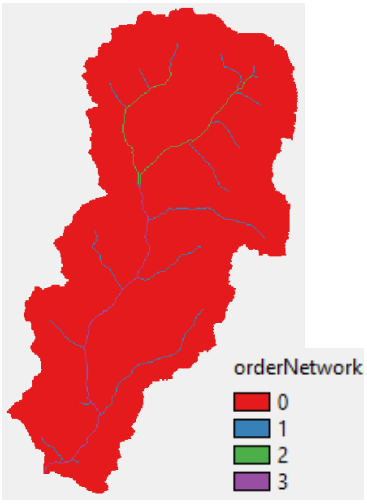
Table 6.2: Example of the resulting 'ReachArrayID' (r.165) and 'SubReachArrayID' (r.201) tables which are similar to 'reachID' but now has been ordered based on the value of Slopelength, needed to order the cells in downstream direction.

(a) ReachArrayID

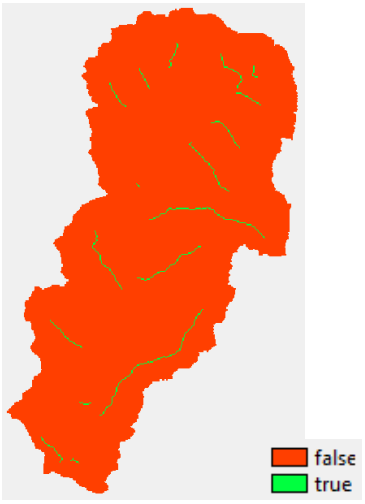
ID	x	y	DEM	Slopelength
1	350	156	4959.8	0.0
2	397	32	707.0	0.0
3	365	67	826.0	0.0
⋮				
665	377	87	616.1	38541.6
666	377	86	600.4	38791.6

(b) SubReachArrayID

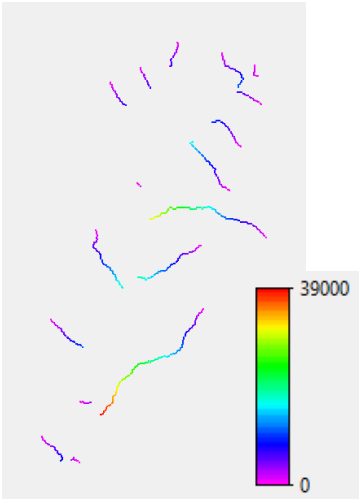
ID	x	y	Slopelength	#order	#sub
1	280	180	0.0	1	1
2	281	179	353.6	1	1
3	282	178	707.1	1	1
⋮					
127	377	87	38541.6	1	1
128	377	86	38791.6	1	1



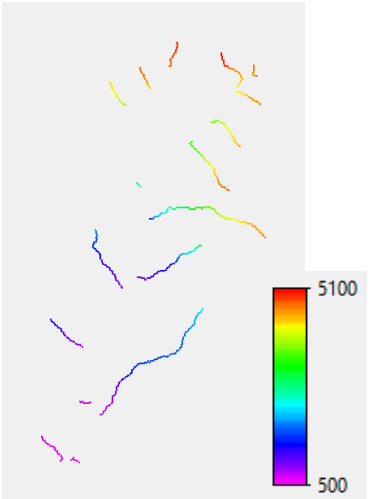
(a) Streamorder determined based on Strahler.



(b) All sub-reaches of order 1



(c) Slopelength of these sub-reaches.



(d) DEM of these sub-reaches

Figure 6.4: Determine the streamorder for all reaches and identify all sub-reaches per streamorder.

6.2 Conservation of mass

So-called 'pits' are created at every outlet and inlet of a reach to withdraw water from the system in order to prevent accounting for the volume of water twice through routing as open water ánd as groundwater through the soil system. With this pit the water is forced out of the system and cannot flow beyond this point. A pit is created by adjusting the local drain direction map at this point, i.e. it is changed into number '5' as this creates a pit by definition. This way the total amount of water in the system is preserved. For a more elaborate explanation consult section 6.5 and see figure 6.8.

6.3 Temporal and spatial grid

The routing module is set up outside the PCRaster environment which provides the opportunity to run the routing module with a different temporal and spatial resolution than the simulation of the SPHY model itself. An advantage of this is that when preferred a finer resolution can be chosen, which improves the quality of the numerical solution. By specifying a smaller time step the numerical solution will approach the analytical solution, however this will increase computational time. Choosing a time step that is significantly larger than the system time scale may lead to instabilities or oscillations. It is therefore a balance between the quality of the numerical approximation and computational time. The temporal scale should also be in balance with the spatial scale to avoid instabilities and to limit excessive numerical diffusion. Time and space problems are coupled through the Courant number (see Eq. 5.1). This number should be smaller than 1 to limit the numerical diffusion. The user receives a warning when the Courant number exceeds 1.

The routing can be calculated on a different temporal resolution than on which the visualisation of the result are done. They are independent of each other, i.e. the routing calculations can be done on an hourly base, while the visualisation can be given on a daily base.

6.4 Initial and boundary conditions

A spatially constant discharge of 25.0 m^3 s^{-1} has been specified as initial condition for all reaches. This has been calculated from rough estimations of width, depth and flow velocity in the Tamakoshi river. At the upstream boundary a flow hydrograph needs to be specified. This comprises the accumulated discharge of the area upstream of this point. At the downstream boundary preferably a known Q-h relation is provided, however in data-scarce areas such as the Tamakoshi river basin this relation is unknown. Therefore, another option is to provide a flow hydrograph as well, similarly to the upstream boundary. However, to strengthen the numerical stability of the system, it was chosen to use a linear combination of the Dirichlet and Neumann boundary conditions as was explained in section 5.3. The flow hydrograph of the outlet of the entire basin comprises the accumulated discharge of the whole catchment as simulated by SPHY. The downstream flow hydrographs of all the other individual reaches of the stream network are determined by applying the cD-equation from downstream to upstream direction. This method will be explained more elaborately in section 6.8.

6.5 Lateral inflow

An artificial experiment has been done to demonstrate the effect of adding lateral inflow at a certain position along a stream. Figure 6.5 shows how an initial impulse to the stream system shifts and attenuates as the wave travels through the channel. Time is depicted along the x-axis time and the three coloured lines correspond to hydrographs simulated at distance $1/4^{th}$, $2/4^{th}$ and $3/4^{th}$ along the stream respectively.

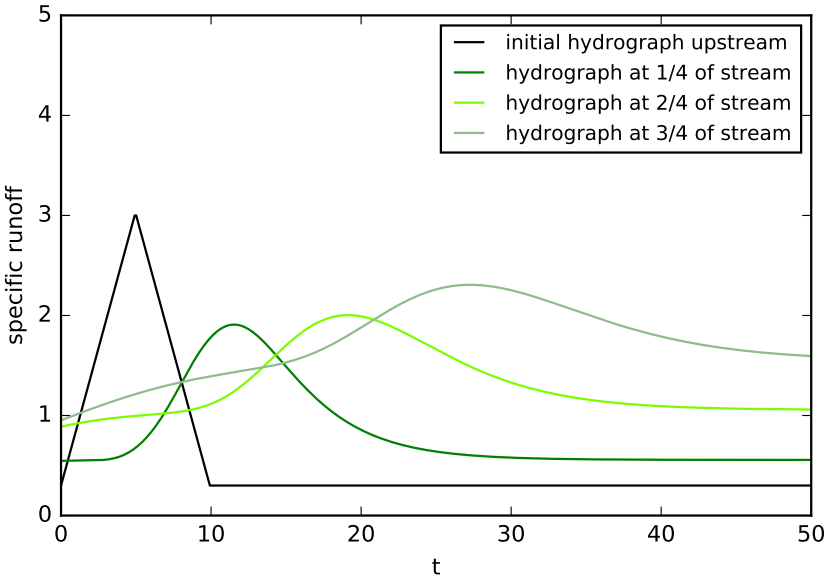


Figure 6.5: Resulting hydrographs at several distances along the stream channel.

Figure 6.6 is the result of adding lateral flow at both $1/4^{th}$ and $2/4^{th}$ of the channel. This affects the hydrographs at these locations instantly.

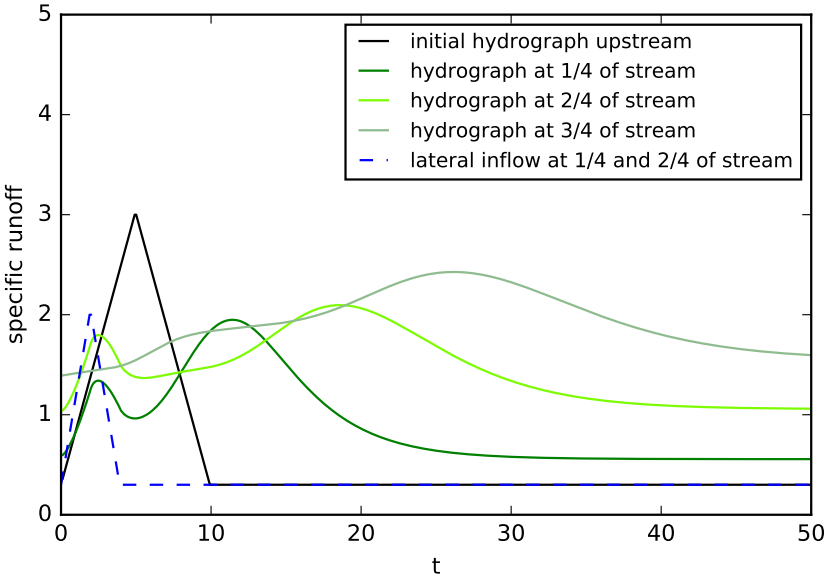


Figure 6.6: Resulting hydrographs at several distances along the stream channel after adding lateral inflow at two places along the channel.

The same has been done in figure 6.7, but now the timing of the impulse of lateral flow at a distance of $2/4^{th}$ of the channel is shifted.

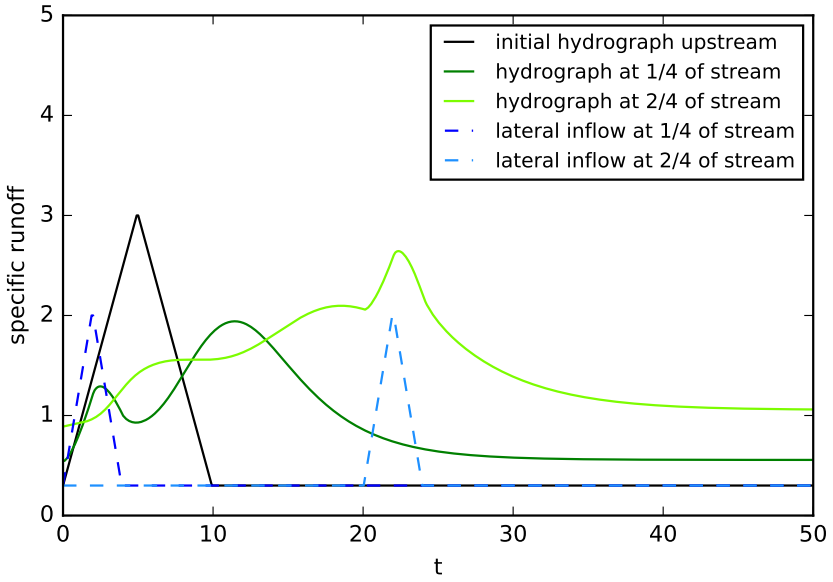


Figure 6.7: Resulting hydrographs at several distances along the stream channel after adding lateral inflow at two places along the channel.

There are two sources of water that together form the lateral inflow for the next reach (see Fig.6.8). The accumulated specific runoff produced by the cells upstream of the inlet of the reach is routed through the channel (dashed blue area). The amount at the outlet of this reach after routing is one source of lateral flow for the next downstream reach ($\text{Water}_{\text{routed}}$). Besides the injection of lateral flow originating from the open channel, an additional flow is added as lateral flow to this point. This water originates from the system and has not been routed yet. This is the total amount of accumulated specific runoff of the cells upstream of the point of injection minus the blue area (i.e. the red area).

Adding the outflow from one reach into the next as lateral inflow can be done using different methods. It can be injected at one single point or it can be divided along a transect. Although knowing that by adding a large volume of water at a single point it can lead to numerical unstable conditions, it has been chosen from a pragmatic point of view to use the point injection here. Choosing a smaller threshold value, and thereby refining the stream network, will decrease the effect of the point injection.

If the threshold to determine the stream network is relatively high, i.e. less cells are assigned to the stream network, the water from the system that is added as lateral flow at one point to the next stream can become considerable large (see Fig.6.8). As a result a large amount of water that has not been routed yet will suddenly be added to the channel at a single point.

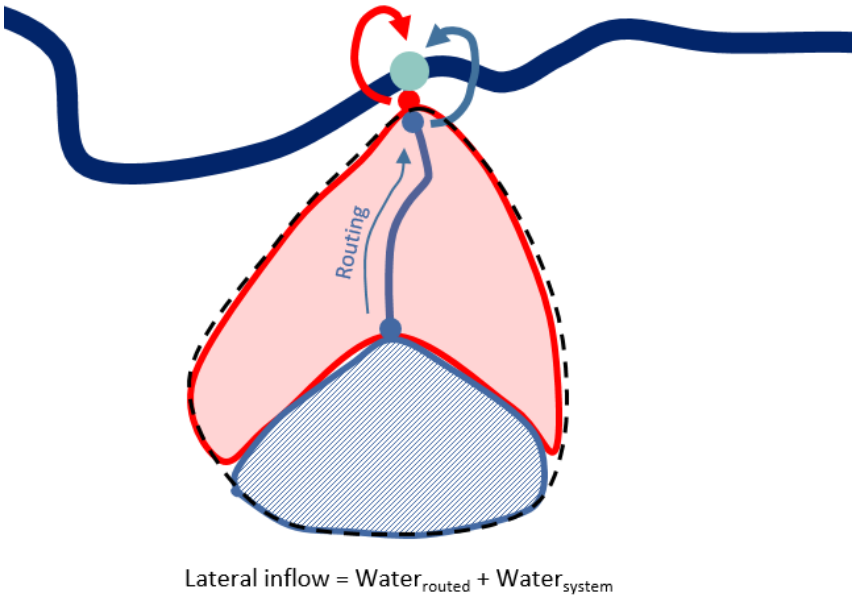


Figure 6.8: Components that together form the lateral inflow to be injected to the next reach at the connection point.

6.6 Parameter values to be specified in config-file

There are several parameters that need to be specified by the user in the SPHY config-file. First the user should specify a threshold value that specifies the minimal area upstream of a cell required for this cell to be assigned to the stream network. This should be provided in km^2. Values for α and β should be provided to specify the weight to be given to the Neumann and Dirichlet part of the boundary condition respectively. A value for ω should be given to specify the Crank-Nicolson scheme to be more implicit or explicit (i.e. $\omega = 1 \rightarrow$ fully implicit, $\omega = 0 \rightarrow$ fully explicit).

Furthermore, as was explained in section 6.3 the calculations in the routing module can be done on a different temporal and spatial resolution. Therefore the user must specify the time step at which the routing calculations are performed, i.e. *dt*, as well as the time step at which the results are provided and visualised, i.e. *dt_visualize*. The user can opt for $^1/_{24}$, $^3/_{24}$, $^6/_{24}$, $^{12}/_{24}$ or $^{24}/_{24}$ daily time steps. The spatial step *dx* should be chosen such that the result of the division: $dx_{clonemap}/dx$ is an integer number, i.e. *dx* can be larger or smaller than the spatial resolution of the clone map that is used. If the user does not specify a value *dx* will equal the spatial resolution of the clone map.

In addition, values for the wave celerity (c) and diffusion coefficient (D) should be provided. The user can opt to directly provide a value for both parameters. The other option is to calculate c and D starting from the linearised convection convection-diffusion equation (see Eq. 6.2 and 6.3). For the latter option a representative width and depth of the river should be provided, as well as the roughness coefficient and bottom slope. For further elaboration on the derivation of the equations consult Torfs (2002). Estimation of these parameters are subject to a number of uncertainties and the results of the calculations of c and D are sensitive to slight changes of these parameters. Therefore most probably better results are obtained when c and D are calibrated.

$$c = \frac{3}{2} S_0^{1/2} C h^{1/2} \tag{6.2}$$

$$D = \frac{C h^{3/2}}{2 S_0^{1/2}} \tag{6.3}$$

Where:

c	:	wave celerity (m s^{-1})
D	:	diffusion coefficient (m^2 s^{-1})
C	:	Chézy coefficient ($m^{1/2}$ s^{-1})
S_0	:	Bed slope (m m^{-1})
h	:	Water depth (m)

These calculations are programmed in the script *cD*:

```
8  def calcCD(self, width, depth, roughness, slope):

10      # Cross sectional area (m2)
11      A=float(width)*float(depth)
12      # Wetted perimeter (m)
13      P=2*depth+width
14      # Hydraulic radius (m)
15      R=A/P

17      # Chezy coefficient (m(1/2)*s-1)
18      Chezy = (1/float(roughness))*(R**(1/6))

20      # Wave celerity
21      c=(3/2.)*(float(slope)**0.5)*Chezy*(float(depth)**0.5)   # m s-1
22      c = c * 60*60)   # m hr-1
23      c = c /1e3)   # km hr-1

25      # Diffusion coefficient
26      D=(Chezy*(float(depth)**(3/2.)))/(2*(float(slope)**0.5))   # m2 s-1
27      D = D * 60*60   # m2 hr-1
28      D = D /1e6   # km2 hr-1

30      return c,D
```

6.7 The cD-equation

The following functions are used to solve the cD-equation (see Sect. 5.3). The former function *runCDback* is called upon first in order to determine the downstream boundary conditions for all individual reaches (see Sect. 6.8) (r.28–103). The latter function *runCD* is used for the main routing loop (see Sect. 6.9) (r.106–189). They are similar except for the fact that in the function *runCD* lateral flow is included and that the reaches are artificially elongated in order to decrease the effect of the imposed downstream boundary conditions (see Sect. 5.2).

```
32 def runCDback(up, down, init, length, dx, dt, c, D):

34     # Length of reach in km
35     L = length

37     # Number of spatial steps
38     Nx = int((L/dx)+1)

40     # Divide the volume of water over smaller time steps when the chosen temporal
41     # step is smaller than 24 hours.
42     if dt < 24:
43         up_smalldt = []
44         for i in range(len(up)-1):
45             t_up = (up[i+1]-up[i])/(24/dt)
46             for j in range(1,(24/dt)+1):
47                 up_smalldt.append(up[i]+t_up*j)
48         up = up_smalldt

50     # Number of time steps
51     Nt=len(up)

53     # Define sigma and rho in the context of numerical discretization
54     sigma = float(D*dt)/float(2.*dx*dx)
55     rho = float(-c*dt)/float(4.*dx)
56     print 'Courant [-] = ' + str(4*-rho)
57     print 'Peclet [-] = ' + str(dx*c/D)
58     print '100 numerical time steps [hr] = ' + str(100*dt)
59     print 'Travel distance in 100 numerical time steps [km] = ' + str(c*100*dt)
60     print '100 numerical spatial steps [km] = ' + str(100*dx)
61     print 'Travel time over 100 numerical spatial steps [hr] = ' + str(100*dx/c)
62     print 'Travel time over 50 km [hr] = ' + str(50./c)

64     # Specify the weight given to Neumann and Dirichlet part of the Robin b.c.
65     alpha = config.getfloat('CDROUTING','alpha')
66     beta = config.getfloat('CDROUTING','beta')

68     # Implicit/explicit (omega=1 --> fully implicit, omega=0 --> fully explicit)
69     omega = config.getfloat('CDROUTING','omega')

71     # Create tridiagonal matrices
72     A = np.diagflat([(-sigma+rho)*omega for i in range(Nx-1)], -1) +
73     np.diagflat([1.+2.*sigma*omega for i in range(Nx)]) +
74     np.diagflat([(-(sigma+rho))*omega for i in range(Nx-1)], 1)
75     A[0,:]  = np.array([1] + [0 for i in range(0,Nx-1)])
76     # If reach is shorter than 4 cells:
77     if Nx<4:
78         A[-1,:]  = np.array([0 for i in range(0,Nx-1)] + [1])
```

```
79      # In all other cases:
80      else:
81          A[-1,:]  = np.array([0 for i in range(0,Nx-3)] + [-1] +
              [(2*beta*dx)/alpha] + [1])

83      B = np.diagflat([(sigma-rho)*(1-omega) for i in range(Nx-1)], -1) +
84      np.diagflat([1.-2.*sigma*(1-omega) for i in range(Nx)]) +
85      np.diagflat([(sigma+rho)*(1-omega) for i in range(Nx-1)], 1)
86      B[0,:]  = np.array([0 for i in range(0,Nx)])
87      B[-1,:]  = np.array([0 for i in range(0,Nx)])

89      # Create vector F
90      f_vec = np.array([0 for i in range(0,Nx)])

92      # Specify initial condition
93      Q = init

95      # Create empty list to store the results
96      Q_record = []

98      # First create a stationary solution which is subsequently provided to the
99      # main loop.  To create this stationarity no impulse is given upstream.
100      for ti in range(1,100):
101          Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
102          Q = Q_new
103      Q_record.append(Q)

105      # Main loop to solve the system iteratively
106      for ti in range(1,Nt):
107          # Provide upstream hydrograph
108          f_vec[0] = up[ti]*dt
109          # Provide downstream hydrograph
110          f_vec[-1] = (2*dx*down[ti])/alpha*dt
111          # Solve the system
112          Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
113          Q = Q_new
114          Q_record.append(Q)
115      Q_record = np.array(Q_record)

117      return Q_record



120  def runCD(up, down, init, lat, check_lat, lat_distr, length, dx, dt, c, D):

122      # Length of reach in km
123      L = length
124      # Artificially elongate the reach for the downstream b.c.  to have less influence

125      L_long = length *1.2

127      # Number of spatial steps
128      Nx = int((L/dx)+1)
129      Nx_long = int((L_long/dx)+1)

131      # Divide the volume of water over smaller time steps when the chosen temporal
```

```
132     # step is smaller than 24 hours.
133     if dt < 24:
134         up_smalldt = []
135         for i in range(len(up)-1):
136             t_up = (up[i+1]-up[i])/(24/dt)
137             for j in range(1,(24/dt)+1):
138                 up_smalldt.append(up[i]+t_up*j)
139         up = up_smalldt

141     # Number of time steps
142     Nt=len(up)

144     # Define sigma and rho in the context of numerical discretization
145     sigma = float(D*dt)/float(2.*dx*dx)
146     rho = float(-c*dt)/float(4.*dx)
147     print 'Courant [-] = ' + str(4*-rho)
148     print 'Peclet [-] = ' + str(dx*c/D)
149     print '100 numerical time steps [hr] = ' + str(100*dt)
150     print 'Travel distance in 100 numerical time steps [km] = ' + str(c*100*dt)
151     print '100 numerical spatial steps [km] = ' + str(100*dx)
152     print 'Travel time over 100 numerical spatial steps [hr] = ' + str(100*dx/c)
153     print 'Travel time over 50 km [hr] = ' + str(50./c)

155     # Specify the weight given to Neumann and Dirichlet part of the Robin b.c.
156     alpha = config.getfloat('CDROUTING','alpha')
157     beta = config.getfloat('CDROUTING','beta')

159     # Implicit/explicit (omega=1 --> fully implicit, omega=0 --> fully explicit)
160     omega = config.getfloat('CDROUTING','omega')

162     # Create tridiagonal matrices
163     A = np.diagflat([(-sigma+rho)*omega for i in range(Nx_long-1)], -1) +
164     np.diagflat([1.+2.*sigma*omega for i in range(Nx_long)]) +
165     np.diagflat([(-(sigma+rho))*omega for i in range(Nx_long-1)], 1)
166     A[0,:]  = np.array([1] + [0 for i in range(0,Nx_long-1)])
167     # If reach is shorter than 4 cells:
168     if Nx_long<4:
169         A[-1,:]  = np.array([0 for i in range(0,Nx_long-1)] + [1])
170     # In all other cases:
171     else:
172         A[-1,:]  = np.array([0 for i in range(0,Nx-3)] + [-1] +
            [(2*beta*dx)/alpha] + [1])

174     B = np.diagflat([(sigma-rho)*(1-omega) for i in range(Nx_long-1)], -1) +
175     np.diagflat([1.-2.*sigma*(1-omega) for i in range(Nx_long)]) +
176     np.diagflat([(sigma+rho)*(1-omega) for i in range(Nx_long-1)], 1)
177     B[0,:]  = np.array([0 for i in range(0,Nx_long)])
178     B[-1,:]  = np.array([0 for i in range(0,Nx_long)])

180     # Create vector F
181     f_vec = np.array([0 for i in range(0,Nx_long)])

183     # Specify initial condition
184     Q = init

186     # Create empty list to store the results
```

```
187     Q_record = []

189     # First create a stationary solution which is subsequently provided to the
190     # main loop.  To create this stationarity no impulse is given upstream.
191     for ti in range(1,100):
192         Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
193         Q = Q_new
194     Q_record.append(Q)

196     # Main loop to solve the system iteratively
197     for ti in range(1,Nt):
198         # Provide upstream hydrograph
199         f_vec[0] = up[ti]*dt
200         # Add lateral inflows
201         for i in range(len(check_lat)):
202             f_vec[int(lat[i][0])-1] = (lat[i][ti] + lat[i][ti+Nt])*dt
203         # Provide downstream hydrograph
204         f_vec[-1] = (2*dx*down[ti])/alpha*dt
205         # Solve the system
206         Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
207         Q = Q_new
208         Q_record.append(Q)
209     Q_record_long = np.array(Q_record)
210     # Cut of the elongated part of the reach
211     Q_record = Q_record_long[:,0:Nx]

213     return Q_record
```

6.8 Determine downstream hydrographs using cD-equation

The cD-equation is applied to every individual reach of the stream network, for which initial and boundary conditions should be provided. For the inlet of a reach the hydrograph that acts as upstream boundary condition is obtained from the table containing total accumulated runoff (see Sect. 6.8.4). This time series is based on simulations of the SPHY model. However, the downstream boundary conditions for each reach are not explicitly specified. In order to define the hydrograph downstream, the best approximation is to use the hydrograph of the cell of the next stream order to which the outlet is connected to. An iterative process is started in which the cD-equation firstly is applied to the highest stream order, coinciding with the main stream. From this the hydrograph of the connecting points to its sub-reaches is retrieved, which act as downstream boundary conditions for the sub-reaches. Subsequently, the cD-equation will be applied to these sub-reaches, from which the hydrograph of the subsub-reaches are obtained. This process continues until all reaches are included. For this process the locations of the inlets and outlets need to be know, as well as the connecting points and which reaches are connecting to each other.

6.8.1 Determine locations of inlets and outlets

In section 6.1 the inlets and outlets of all reaches were determined. Continuing on that, the locations (i.e. row and column in the raster) of all outlets and all inlets are stored in 'pitID' and 'inID' respectively. In addition 'uphydroID' contains the locations of the inlets of all reaches with stream order > 1, and in 'inoutID' the locations of all the outlets together with the inlets of the reaches of the smallest stream order are stored. For further analysis it is needed to convert the array of the adjusted local drain direction (i.e. containing pits) back to a PCRmap, as well as converting the array with the inlets to a PCRmap.

```
262 # Reshape the arrays to 4 columns.
263 # Save the arrays containing 'order', 'sub' and 'x and y'
264 outIDreach=numpy.reshape(outIDreach, (len(outIDreach)/4,4))
265 inIDreach=numpy.reshape(inIDreach, (len(inIDreach)/4,4))
266 upsIDreach=numpy.reshape(upsIDreach, (len(upsIDreach)/4,4))
267 numpy.savetxt(outpath + 'outIDreach.txt', outIDreach, delimiter=',')
268 numpy.savetxt(outpath + 'inIDreach.txt', inIDreach, delimiter=',')
269 numpy.savetxt(outpath + 'upsIDreach.txt', upsIDreach, delimiter=',')

271 # Convert ldd array 'ldd_inout' back to pcr-map.  This map contains pits at the
272 # inlets of the reaches of the smallest streamorder and additionally pits at all outlets.
273 ldd_upd = pcr.numpy2pcr(Ldd, ldd_inout, MV)
274 pcr.report(ldd_upd, outpath + 'ldd_upd.map')
275 # Convert inlet array to pcr-map (inlets of streamorder 1)
276 inlets = pcr.numpy2pcr(Boolean, ldd_in, MV))
277 pcr.report(inlets, outpath + 'inlets.map')
278 # Convert outlet array to pcr-map (outlets of all streamorders)
279 outlets = pcr.numpy2pcr(Boolean, ldd_out, MV)
280 pcr.report(outlets, outpath + 'outlets.map')
281 # Convert inlets of streamorder>1 array to pcr-map
282 ups = pcr.numpy2pcr(Boolean, ldd_ups, MV)
283 pcr.report(ups, outpath + 'ups.map')

285 # Determine locations of only all the outlets
286 pitID = numpy.argwhere(ldd_out==1)
287 numpy.savetxt(outpath + 'pitID.txt', pitID, delimiter=',')
288 # Determine locations of only all the inlets of streamorder 1
289 inID = numpy.argwhere(ldd_in==1)
290 numpy.savetxt(outpath + 'inID.txt', inID, delimiter=',')
291 # Determine locations of inlets of all stream orders > 1
292 # Needed to have a first estimation of the upstream b.c.  when runCDback.
293 uphydroID = numpy.argwhere(ldd_ups==1)
```

```
294 numpy.savetxt(outpath + 'uphydroID.txt', uphydroID, delimiter=',')
295 # Determine locations of only all the outlets plus inlets of streamorder 1
296 inoutID = numpy.argwhere(ldd_inout==5)
297 numpy.savetxt(outpath + 'inoutID.txt', inoutID, delimiter=',')
```

Table 6.3: The different 'IDreach' tables contain information about the locations of e.g. the inlets or outlets of each reach (r.267–269). The different 'ID' tables contain similar information as the 'IDreach' tables, however now in the order that PCRaster scans the raster (r.287, 290, 294 and 297). 'connectID' shows the location where a stream flows into the next stream (r.328).

(a) out/in/upsIDreach

#order	#sub	x	y
1	1	280	180
1	2	214	237
1	3	222	178
⋮			
2	2	58	150
3	1	165	123

(b) pit/in/uphydro/inoutID

x	y
35	156
45	197
56	227
⋮	
397	32
421	66

(c) connectID

x	y
58	150
58	150
65	225
⋮	
419	58
421	48

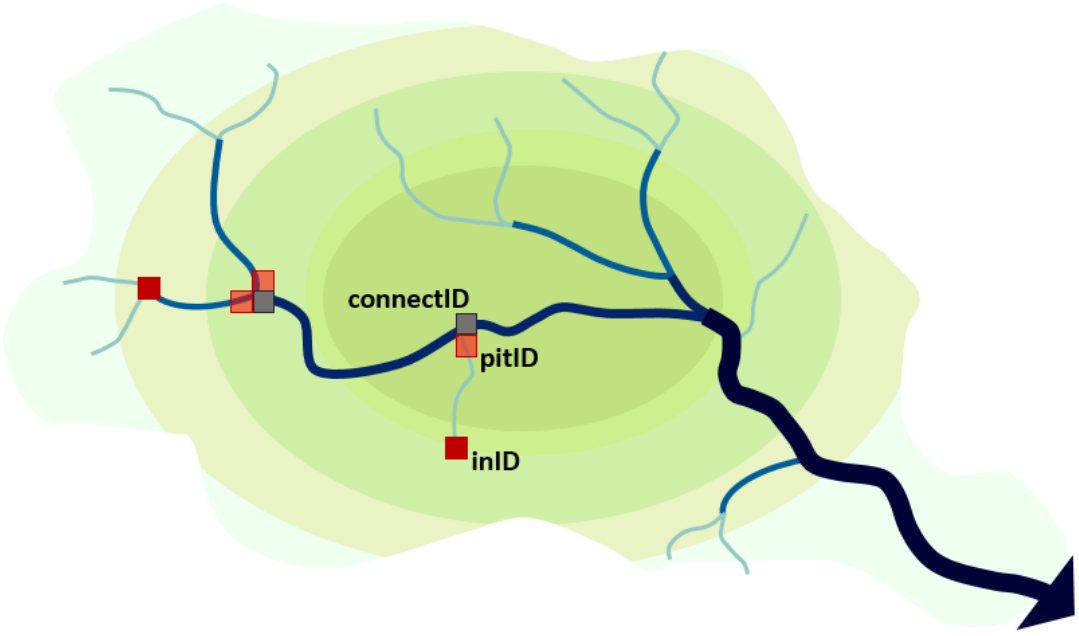


Figure 6.9: River network with examples of the inlet-, outlet- and connected grid cell visualised.

6.8.2 Determine connecting points

To find the points where two reaches are connected to each other the local drain direction code has been used. For every cell the local drain direction map assigns an arrow pointing to its downstream neighbouring cell, which is linked to a value from 1 to 9 according to figure 6.10. Starting from the location of the outlet the connecting point can be determined using this local drain direction by adding or subtracting a row and/or column.

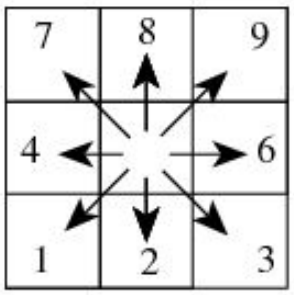


Figure 6.10: Codes linked to the local drain direction.

```
299 # Finding the connecting point to the stream of the next order using ldd-direction
300 ldd_array = pcr.pcr2numpy(FlowDir_ini, MV)
301 connectID=numpy.zeros((len(pitID),2)) # Create empty array with 2 columns
302 for i in range(0,len(pitID)):
303     direction = ldd_array[pitID[i][0]][pitID[i][1]]
304     if direction == 1:
305         connectID[i,0] = pitID[i][0]+1
306         connectID[i,1] = pitID[i][1]-1
307     if direction == 2:
308         connectID[i,0] = pitID[i][0]+1
309         connectID[i,1] = pitID[i][1]
310     if direction == 3:
311         connectID[i,0] = pitID[i][0]+1
312         connectID[i,1] = pitID[i][1]+1
313     if direction == 4:
314         connectID[i,0] = pitID[i][0]
315         connectID[i,1] = pitID[i][1]-1
316     if direction == 6:
317         connectID[i,0] = pitID[i][0]
318         connectID[i,1] = pitID[i][1]+1
319     if direction == 7:
320         connectID[i,0] = pitID[i][0]-1
321         connectID[i,1] = pitID[i][1]-1
322     if direction == 8:
323         connectID[i,0] = pitID[i][0]-1
324         connectID[i,1] = pitID[i][1]
325     if direction == 9:
326         connectID[i,0] = pitID[i][0]-1
327         connectID[i,1] = pitID[i][1]+1
328 numpy.savetxt(outpath + 'connectID.txt', connectID, delimiter=',')
```

6.8.3 Locate connecting reaches

The following programming code is used to determine which reaches are connecting to each other:

```
333 # Create empty list.
334 coupled_reaches=[]
335 for i in range(0,len(outIDreach)):
336     # Find where in pitID the first entry of outIDreach is located
337     index = numpy.where(numpy.logical_and(pitID[:,0]==outIDreach[i,2],
        pitID[:,1]==outIDreach[i,3]))[0][0]
338     # At this same index in connectID the corresponding connecting point to
        # this pit can be found
339     row=connectID[index][0]
340     col=connectID[index][1]
341     # Find the order that is one higher than the one considered
342     order=outIDreach[i,0]+1
343     # Determine the number of sub-reaches that order has
344     Nreaches=inIDreach[:,0].tolist().count(float(order))

346     # Continue this loop until the corresponding connecting reach is found
347     # or when all reaches have been regarded
348     sub=1
349     found='false'
350     while found!='true':
351         if order>strordmax:
352             found='true'
353         else:
354             # Read file containing the locations of the sub-reaches
                # of the higher order
355             reachIDs = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order)
                + '_' + str(sub) + '.txt', delimiter=',')
356             # If reach consists of only 1 cell
357             if len(reachIDs)/6.==1:
358                 # Check whether reachIDs contains the connecting cell that is considered
359                 # If that is TRUE, then store the number of the order and the sub-reach
360                 # of the reaches that are connected to each other
361                 # If FALSE, check the next sub-reach
362                 a = (numpy.logical_and(reachIDs[1]==float(row),
                    reachIDs[2]==float(col))).tolist()
363                 if a:
364                     found = 'true'
365                     coupled_reaches.append(int(outIDreach[i,0]))
366                     coupled_reaches.append(int(outIDreach[i,1]))
367                     coupled_reaches.append(int(order))
368                     coupled_reaches.append(int(sub))
368                 else:
370                     found='false'
371                     sub+=1
372                     if sub==Nreaches+1:
373                         order+=1
374                         sub=1
375                         Nreaches=inIDreach[:,0].tolist().count(float(order))

377             # If reach consists of more than 1 cell
378             else:
379                 # Check whether reachIDs contains the connecting cell that is considered
```

```
380                  # If that is TRUE, then store the number of the order and the sub-reach
381                  # of the reaches that are connected to each other
382                  # If FALSE, check the next sub-reach
383                  a = (numpy.logical_and(reachIDs[:,1]==float(row),
                    reachIDs[:,2]==float(col))).tolist()
384                  if a.count(True)==1:
385                      found = 'true'
386                      coupled_reaches.append(int(outIDreach[i,0]))
387                      coupled_reaches.append(int(outIDreach[i,1]))
388                      coupled_reaches.append(int(order))
389                      coupled_reaches.append(int(sub))
390                  else:
391                      found='false'
392                      sub+=1
393                      if sub==Nreaches+1:
394                          order+=1
395                          sub=1
396                          Nreaches=inIDreach[:,0].tolist().count(float(order))

398  # Reshape the list.
399  # 1st column:order of sub-reach, 2nd column:sub of sub-reach,
400  # 3rd column:order to which sub-reach drains (=order of reach), 4th column:sub of reach
401  coupled_reaches=numpy.reshape(coupled_reaches, (len(coupled_reaches)/4,4))
402  numpy.savetxt(outpath + 'coupled_reaches.txt', coupled_reaches, delimiter=',')
```

An example of the format of the resulting table is shown in table 6.4:

Table 6.4: Example of the resulting table showing which reaches are coupled to each other (r.402).

Order upstream	Sub-reach upstream	Order downstream	Sub-reach downstream
1	1	2	1
⋮			
1	4	2	2
1	5	2	1
⋮			
2	1	3	1
2	2	3	2
2	3	4	1
2	4	3	1
2	5	3	2
3	1	4	1
3	2	4	1

6.8.4 Accumulated runoff at inlets and outlets

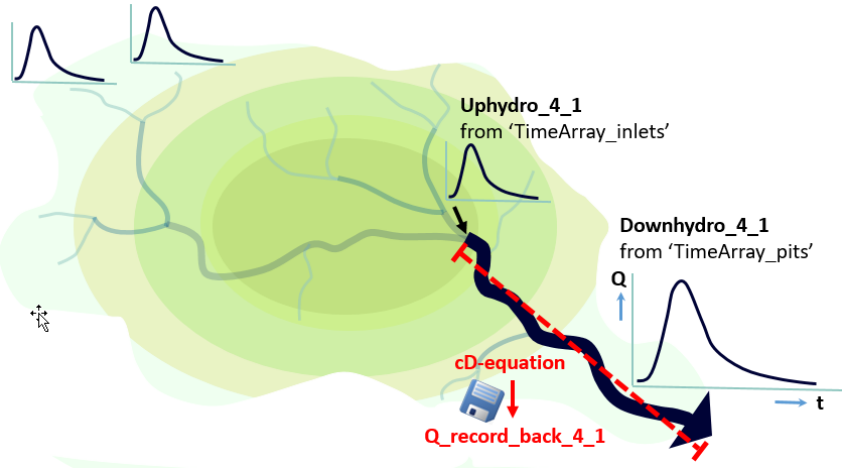
The following code is used to determine the flow hydrograph for each pit (*TimeArrayPit*) and inlet of stream order 1 (*TimeArrayIn*) as simulated by the SPHY model without routing. A separate table is created to store these hydrographs of only the inlets of all reaches with stream order > 1 (TimeArrayUphydro), as well as for the hydrograph of the outlet of the entire catchment (*TimeArrayOutlet*). Discharge is given in $m^3\ s^{-1}$.

```
435 ##########
436 # Determine daily accumulated runoff for each pit and inlet for all timesteps
437 ##########
438 timestep = (enddate-startdate).days+1
439 # Create empty arrays that have a number of rows equal to the number of timesteps,
440 # and have a number of columns equal to the amount of outlets/inlets respectively
441 TimeArrayPit = numpy.zeros(((timestep+1),(len(pitID)+1)))
442 TimeArrayIn = numpy.zeros(((timestep+1),(len(inID)+1)))
443 TimeArrayUphydro = numpy.zeros(((timestep+1),(len(uphydroID)+1)))
444 TimeArrayOutlet = []

446 # For-loop over all timesteps
447 for time in range(1,timestep+1):
448     # Read the runoff time-series
449     runoff = pcr.readmap(pcrm.generateNameT(Runoff, time))
450     runoff = pcr.ifthen(clone, runoff*0.001*pcr.cellarea()/(24*3600.))
452     # Create map
453     pcr.report(runoff, outpath + 'runoff_' + str(time) + '.map')

455     # Calculate the accuflux of runoff for each pit and inlet and
        # write it to numpy array
456     accu_pit = pcr.ifthen(ldd_upd==5,pcr.accuflux(ldd_upd,runoff))
457     pcr.report(accu_pit, outpath + 'accu_pit.map')
458     accu_in = pcr.ifthen(inlets==1, pcr.accuflux(ldd_upd,runoff))
459     pcr.report(accu_in, outpath + 'accu_in.map')
460     accu_up = pcr.ifthen(ups==1, pcr.accuflux(FlowDir_ini,runoff))

462     # Convert PCRmap to numpy array
463     runoff_array_pit = pcr.pcr2numpy(accu_pit,MV)
464     runoff_array_in = pcr.pcr2numpy(accu_in,MV)
465     runoff_array_ups = pcr.pcr2numpy(accu_up,MV)

467     # Fill the time array with time series of the accuflux of runoff for all pits
468     for j in range(0,len(pitID)):
469         TimeArrayPit[0,0] = MV
470         TimeArrayPit[0,j+1] = j+1 # first row shows the pit number
471         TimeArrayPit[time,0] = time # first column shows timestep
472         # find amount of total runoff per pit
473         TimeArrayPit[time,j+1] = runoff_array_pit[pitID[j][0]][pitID[j][1]]

475     # Fill the time array with time series of the accuflux of runoff for all inlets
        # of streamorder 1
476     for j in range(0,len(inID)):
477         TimeArrayIn[0,0] = MV
478         TimeArrayIn[0,j+1] = j+1 # first row shows the inlet number
479         TimeArrayIn[time,0] = time # first column shows timestep
480         # find amount of total runoff per inlet
481         TimeArrayIn[time,j+1] = runoff_array_in[inID[j][0]][inID[j][1]]
```

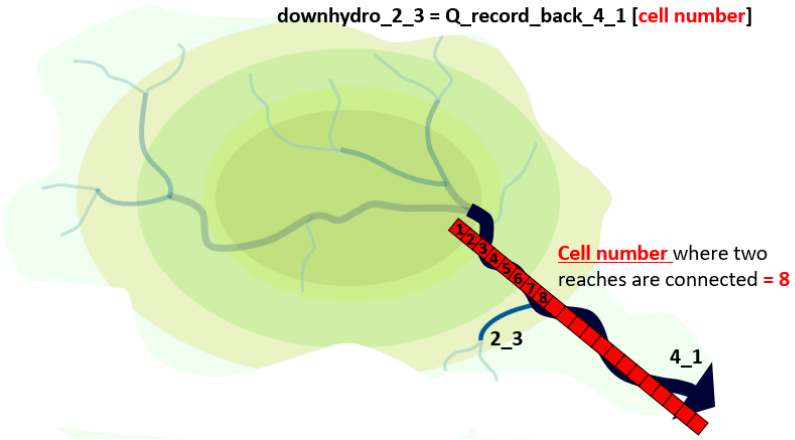
```
483      # Fill the time array with time series of the accuflux of runoff for all inlets
484      for j in range(0,len(uphydroID)):
485           TimeArrayUphydro[0,0] = MV
486           TimeArrayUphydro[0,j+1] = j+1 # first row shows the inlet number
487           TimeArrayUphydro[time,0] = time # first column shows timestep
488           # Find amount of total runoff per inlet
489           TimeArrayUphydro[time,j+1] = runoff_array_ups[uphydroID[j][0]][uphydroID[j][1]]

491      # Calculate the accuflux of the runoff for the outlet of the total area
492      # using the original ldd-map
493      accu_outlet = pcr.accuflux(FlowDir_ini,runoff)
494      runoff_outlet = pcr.mapmaximum(accu_outlet)
495      runoff_outlet = pcr.pcr2numpy(runoff_outlet,MV)
496      runoff_outlet = runoff_outlet[0][0]
497      TimeArrayOutlet.append(runoff_outlet)

499  # Save the time arrays
500  numpy.savetxt(outpath + 'TimeArray_pits.txt', TimeArrayPit, delimiter=',')
501  numpy.savetxt(outpath + 'TimeArray_inlets.txt', TimeArrayIn, delimiter=',')
502  numpy.savetxt(outpath + 'TimeArray_ups.txt', TimeArrayUphydro, delimiter=',')
503  numpy.savetxt(outpath + 'TimeArray_outlet.txt', TimeArrayOutlet, delimiter=',')
```

Table 6.5: The 'TimeArrays' store for each location of interest the accumulated runoff in $m^3\ s^{-1}$ at that point for every timestep (r.500–503).

Timestep	location 1	location 2	...	location N
1	0	0	...	0.010
2	0.034	0.028	...	0.304
3	0.052	0.044	...	0.328
⋮				
T-1	1.813	2.045	...	0.741
T	0.943	1.215	...	0.636



(a) Run cD-equation for most downstream reach (highest stream order).



(b) Retrieve 'downhydro' from 'Q_record' for connecting upstream reaches.

Figure 6.11: Retrieve 'uphydro' and 'downhydro' for the reach of maximum streamorder from the time series of accumulated runoff (stored in TimeArray_inlets and TimeArray_pits respectively). These serve as input for the cD-equation of which the results are stored in 'Q_record_back' (Fig.(a)). Determine at which cell number the reaches are connected, in order to retrieve the time series for 'downhydro' from 'Q_record_back' (Fig.(b)).

6.8.5 Downstream boundary conditions

Now the iterative process is started in which the cD-equation is applied to all reaches (r.667), starting at the main stream (Fig. 6.11) and working in upstream direction (Fig. 6.12). This is done to approximate the downstream boundary condition of each reach since this is the best approximation that can be done with the information available.

```
516 # Calculate cD-equation for every reach of the stream network to determine downstream
517 # boundary conditions, starting at the main reach (highest stream order), and continue
518 # the calculation towards the smaller branches.
519 print 'Starting cD-equation from down- to upstream to determine the conditions
    at the downstream point of each reach.'
520 for order in range(strordmax,strordmin-1,-1):
521     # Number of sub-reaches of the order (by counting the number of rows that
522     # have the same order)
523     Nsubs_this=inIDreach[:,0].tolist().count(float(order))
```

```
525     # Determine Q_record_back for the highest streamorder
526     if order==strordmax:
527         print order
528         # Determine location of inlet
529         rowIDin = upsIDreach[numpy.where(numpy.logical_and(upsIDreach[:,0]==float(order),
            upsIDreach[:,1]==float(1)))[0][0]][2]
530         colIDin = upsIDreach[numpy.where(numpy.logical_and(upsIDreach[:,0]==float(order),
            upsIDreach[:,1]==float(1)))[0][0]][3]
531         # Find where in uphydroID (= which row) the regarded inlet is located
532         indexIN = numpy.where(numpy.logical_and(uphydroID[:,0]==rowIDin,
            uphydroID[:,1]==colIDin))[0][0]
533         # Select the timeseries from TimeArrayUphydro
534         uphydro_maxorder = TimeArrayUphydro[1:,indexIN+1]
535         numpy.savetxt(outpath + 'uphydro_' + str(order) + '_' + str(1)+'.txt',
            uphydro_maxorder, delimiter=',')

537         # Determine location of outlets/pits
538         rowIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]==float(order),
            outIDreach[:,1]==float(1)))[0][0]][2]
539         colIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]==float(order),
            outIDreach[:,1]==float(1)))[0][0]][3]
540         # Find where in pitID (= which row) the regarded pit is located
541         indexOUT = numpy.where(numpy.logical_and(pitID[:,0]==rowIDout,
            pitID[:,1]==colIDout))[0][0]
542         # Select the timeseries from TimeArrayOutlet
543         downhydro_maxorder = TimeArrayOutlet
544         # If dt<24:  divide every timestep by 24/dt
545         if dt < 24:
546             down_smalldt = []
547             for i in range(len(downhydro_maxorder)-1):
548                 t_down = (downhydro_maxorder[i+1]-downhydro_maxorder[i])/(24/dt)
549                 for j in range(1,(24/dt)+1):
550                     down_smalldt.append(downhydro_maxorder[i]+t_down*j)
551             downhydro_maxorder = down_smalldt
552         numpy.savetxt(outpath + 'downhydro_' + str(order) + '_' + str(1)+'.txt',
            downhydro_maxorder, delimiter=',')

554         # Determine number of timesteps
555         Nt = len(downhydro_maxorder)

557         # Read the locations of the cells belonging to the main reach
558         reachIDs = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order) +
            '_' + str(1) + '.txt', delimiter=',')
559         # Length of stream in km (= number of cells * spatial resolution [m])
560         if dx>SpaceRes:
561             if len(reachIDs)%2==0:
562                 length = (len(reachIDs)*grid_res)
563             else:
564                 # Artificially elongate reach
565                 length = ((len(reachIDs)+1)*grid_res)
566         else:
567             length = (len(reachIDs)*grid_res)
568         Nx = int((length/dx)+1)
569         init = numpy.array([base for i in range(0,Nx)])
```

```
571         # Run cD-equation for reach of highest stream order and store this as Q_record_back
572         Q_record_back_maxorder = cD(uphydro_maxorder, downhydro_maxorder,
            init, length, dx, dt, c, D)
573         numpy.savetxt(outpath + 'Q_record_back_'+str(order) + '_' + str(1)+
            '.txt', Q_record_back_maxorder, delimiter=',')

575     # Consider all sub-reaches that flow into this orders reach
576     for sub in range(1,Nsubs_this+1):
577         # Keeping track which reach is considered during the loop
578         print 'order:  '  + str(order)
579         print 'sub:  '  + str(sub)

581         # Find in 'coupled_reaches' which sub-reaches flow into this orders reach
582         subs_upstream = numpy.where(numpy.logical_and(coupled_reaches[:,2]==order,
            coupled_reaches[:,3]==sub))[0]
583         # Stop in case the reach has no sub-reaches
584         if len(subs_upstream)==0:
585             print 'reach has no reaches draining into it.'
586         else:
587             # Special case when regarding the main stream (highest stream order)
588             if order==strordmax:
589                 Q_record_back_order = Q_record_back_maxorder
590             # In all other cases:
591             else:
592                 Q_record_back_order = numpy.loadtxt(outpath + 'Q_record_back_' +
                    str(order) + '_' + str(sub) + '.txt', delimiter=',')

594             # Read the locations of the cells belonging to the reach that is considered
595             reachIDs = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order) +
            '_' + str(sub) + '.txt', delimiter=',')

597             # For-loop over all sub-reaches of this order
598             for i in subs_upstream:
599                 # Find the order- and sub number of the sub-reach upstream of the
                    # considered reach
600                 subsub_order = coupled_reaches[i,0]
601                 subsub_sub = coupled_reaches[i,1]
602                 print 'sub-reach order:  '  + str(subsub_order),
                    ', sub-reach sub:  '+ str(subsub_sub)

604                 # Calculate the following as long as order is not equal to the
                    # smallest stream order
605                 if order!=strordmin:
606                     if subsub_order==strordmin:
607                         # Determine location of inlets
608                         rowIDin = inIDreach[numpy.where(numpy.logical_and(inIDreach[:,0]==
                            float(subsub_order), inIDreach[:,1]==float(subsub_sub)))[0][0]][2]
609                         colIDin = inIDreach[numpy.where(numpy.logical_and(inIDreach[:,0]==
                            float(subsub_order), inIDreach[:,1]==float(subsub_sub)))[0][0]][3]
610                         # Find where in inID (= which row) the
                            # regarded inlet is located
611                         indexIN = numpy.where(numpy.logical_and(inID[:,0]==rowIDin,
                            inID[:,1]==colIDin))[0][0]
612                         # Select the timeseries from TimeArrayIn
613                         uphydro = TimeArrayIn[1:,indexIN+1]
614                         numpy.savetxt(outpath + 'uphydro_' + str(subsub_order) + '_'
```

```
                    + str(subsub_sub)+'.txt', uphydro, delimiter=',')
615                 else:
616                     # Determine location of inlets
617                     rowIDin = upsIDreach[numpy.where(numpy.logical_and(upsIDreach[:,0]==
                        float(subsub_order), upsIDreach[:,1]==float(subsub_sub)))[0][0]][2]
618                     colIDin = upsIDreach[numpy.where(numpy.logical_and(upsIDreach[:,0]==
                        float(subsub_order), upsIDreach[:,1]==float(subsub_sub)))[0][0]][3]
619                     # Find where in uphydroID (= which row) the
                        # regarded inlet is located
620                     indexIN = numpy.where(numpy.logical_and(uphydroID[:,0]==rowIDin,
                        uphydroID[:,1]==colIDin))[0][0]
621                     # Select the timeseries from TimeArrayUphydro
622                     uphydro = TimeArrayUphydro[1:,indexIN+1]
623                     numpy.savetxt(outpath + 'uphydro_' + str(subsub_order) + '_'
                        + str(subsub_sub)+'.txt', uphydro, delimiter=',')

625                 # Determine location of outlets/pits of the 'smaller'-order reaches
                    # --> needed to determine connectID
626                 rowIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]==
                    float(subsub_order), outIDreach[:,1]==float(subsub_sub)))[0][0]][2]
627                 colIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]==
                    float(subsub_order), outIDreach[:,1]==float(subsub_sub)))[0][0]][3]
628                 # Find where in pitID (= which row) the regarded pit is located
629                 indexOUT = numpy.where(numpy.logical_and(pitID[:,0]==rowIDout,
                    pitID[:,1]==colIDout))[0][0]

631                 # Find the how many'd cell (from upstream) of the reach forms

632                 # the connecting cell to the other reach.  This is necessary
633                 # to know from which cell of Q_record_back as calculated
634                 # in cD-equation to get the flow hydrograph
635                 index_streamcell = numpy.where(numpy.logical_and(reachIDs[:,1]==
                    connectID[indexOUT][0], reachIDs[:,2]==connectID[indexOUT][1]))
636                 cellnumber = index_streamcell[0][0] + 1
637                 # Determine cell number when original spatial resolution of the
                    # model differs from spatial resolution used in cD-equation
638                 cellnumber = cellnumber * (grid_res/dx)
639                 numpy.save(outpath + 'cellnumber_' + str(subsub_order) +
                    str(subsub_sub) + str(order) + str(sub), cellnumber)
640                 # Get flow hydrograph of the connecting point and use that as
641                 # downstream b.c.  of the reach of the stream order more upstream
                    # when running the cD-equation for that reach
642                 connecthydro=[]
643                 for i in range(len(Q_record_back_order)):
644                     connecthydro.append(Q_record_back_order[i][cellnumber])
645                 numpy.savetxt(outpath + 'downhydro_' + str(subsub_order) +
                      '_' + str(subsub_sub)+'.txt', connecthydro, delimiter=',')

647                 # Determine length of the regarded subreach of the 'smaller'
order
648                 # Length of stream in km (= number of cells * spatial resolution
[m])
649                 lengthsubsub = numpy.loadtxt(outpath + 'SubReachArrayID_' +
                    str(subsub_order) + '_' + str(subsub_sub) + '.txt', delimiter=',')
650                 if numpy.size(lengthsubsub)==6:
651                     length = grid_res
```

```
652                         if dx>SpaceRes:
653                             # Artificially elongating a 1-cell reach
654                             length = grid_res*(dx/SpaceRes)
655                     else:
656                         if dx>SpaceRes:
657                             if len(lengthsubsub)%2==0:
658                                 length = (len(lengthsubsub)*grid_res)
659                             else:
660                                 length = ((len(lengthsubsub)+1)*grid_res)
661                         else:
662                             length = (len(lengthsubsub)*grid_res)
663                     Nx = int((length/dx)+1)
664                     init = numpy.array([base for i in range(0,Nx)])

666                     # Run cD-equation for the subreach and save the results in Q_record_back
667                     Q_record_back_sub = cD(uphydro, connecthydro, init,
                        length, dx, dt, c, D)
668                     numpy.savetxt(outpath + 'Q_record_back_'+str(subsub_order) +
                        '_' + str(subsub_sub)+'.txt', Q_record_back_sub, delimiter=',')

670                 else:  print 'end'
```

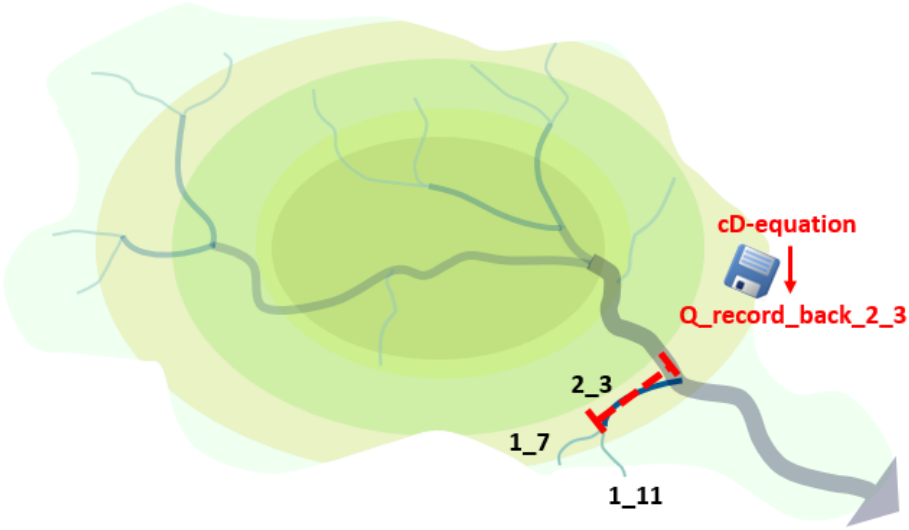
Table 6.6: 'Q_record_back' and 'Q_record' store the results of the cD-equation for each cell of a reach for every timestep (r.668 & 812). '_back' refers to running the cD-equation in upstream direction in order to provide the best possible estimation of the downstream boundary hydrographs for the individual reaches (see Sect. 6.8.5). The number of columns equals the number of cells of the considered reach and the number of rows equals the number of time steps.

(a) Q_record_back

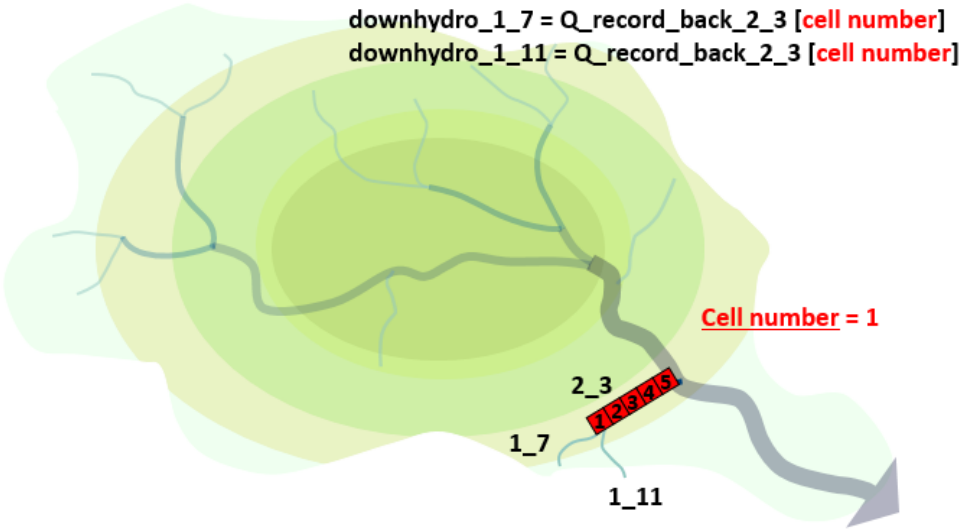
Timestep	cell 1	cell 2	...	cell N
1	0.025	0.004	...	0.652
2	0.034	0.052	...	0.158
3	0.138	0.253	...	0.089
⋮				
T-1	0.338	2.442	...	0.677
T	0.502	2.021	...	0.824

(b) Q_record

Timestep	cell 1	cell 2	...	cell N
1	0.520	0	...	0.520
2	0.682	0.038	...	0.601
3	0.692	0.021	...	0.639
⋮				
T-1	0.891	2.852	...	0.474
T	0.744	2.954	...	0.487



(a) Run cD-equation for the reaches of second highest stream order. Continue this process in upstream direction.



(b) Retrieve 'downhydro' from 'Q_record' for connecting upstream reaches.

Figure 6.12: Retrieve 'uphydro' from the time series of accumulated runoff (stored in TimeArray_inlets). 'Downhydro' is retrieved from time series stored in 'Q_record_back' of the reach this stream flows into. These serve as input for the cD-equation of which the results are stored in 'Q_record_back' (Fig.(a)). Determine at which cell number the reaches are connected, in order to retrieve the time series for 'downhydro' from 'Q_record_back' (Fig.(b)).

Table 6.7: 'Uphydro' and 'Downhydro' store the hydrographs at the inlet and outlet of a reach respectively. The number of rows equals the number of time steps.

(a) Uphydro

Timestep	inlet
1	0.125
2	0.0
3	0.018
⋮	
T-1	1.122
T	1.284

(b) Downhydro

Timestep	outlet
1	1.839
2	1.758
3	1.615
⋮	
T-1	0.625
T	0.784

6.9 Apply cD-equation for routing

Now the final iterative process is started in which the cD-equation is applied to all reaches (r.811), starting at the smallest branches and working in downstream direction. Using this flow routing sequence we assume no backwater effects, which is a valid assumption in an area with substantial elevation differences. The outflow of an upstream reach is injected as lateral flow all at one point where the reach is connected to its downstream branch (r.728–798). Additionally, the accumulated runoff produced in the sub-catchment of the main outlet only (i.e. the runoff produced in the sub-catchments of all other reaches are subtracted) is added as distributed lateral inflow to the main stream (r.800–808). If visualisation of the results are preferred all data is stored in one table *Qall* which is a preparatory step for creating maps of the results (see Sect. 6.10) (r.821–874).

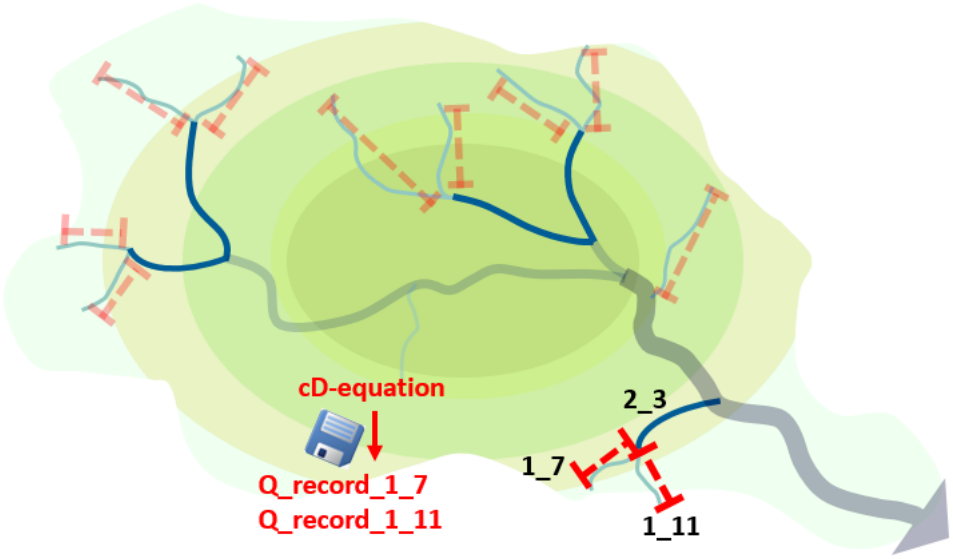
```
675 # Finally calculate cD-equation for every reach of the stream network, starting at
676 # the smallest reaches and working towards the main reach
677 print 'Starting cD-equation from up- to downstream for the entire stream network.'
678 print 'This is the main routing loop.'
679 # Create empty array to which Q_record is concatenated
680 Qall=numpy.empty([len(downhydro_maxorder),0])
681 # Start main loop
682 for order in range(strordmin,strordmax+1):
683     print 'order:  '  + str(order)
684     # Number of sub-reaches of the order (by counting the number of rows that have
685     # the same order)
686     Nsubs_this=inIDreach[:,0].tolist().count(float(order))

688     # Consider all sub-reaches that flow into this orders reach)
689     for sub in range(1,Nsubs_this+1):
690         # Keeping track which reach is considered during the loop
691         print 'sub:  '  + str(sub)

693         # Find in 'coupled_reaches' to which this reach drains
694         sub_downstream = numpy.where(numpy.logical_and(coupled_reaches[:,0]==order,
            coupled_reaches[:,1]==sub))[0]
695         if len(sub_downstream)>0:
696             order_drain = coupled_reaches[sub_downstream[0],2]
697             sub_drain = coupled_reaches[sub_downstream[0],3]

699         # Read the locations of the cells belonging to the reach that is considered
700         reachIDs = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order)
            + '_' + str(sub) + '.txt', delimiter=',')
```

```
        # Determine length of the regarded subreach of the 'smaller' order
        # Length of stream in km (= number of cells * spatial resolution [m])
        if numpy.size(reachIDs)==6:
            length = grid_res
            if dx>SpaceRes:
                # Artificially elongating a 1-cell reach
                length = grid_res*(dx/SpaceRes)
        else:
            if dx>SpaceRes:
                if len(reachIDs)%2==0:
                    length = (len(reachIDs)*grid_res)
                else:
                    length = ((len(reachIDs)+1)*grid_res)
            else:
                length = (len(reachIDs)*grid_res)

        # For streams of the lowest streamorder:  Read upstream hydrograph.
        uphydro = numpy.loadtxt(outpath + 'uphydro_' + str(order)
        + '_' + str(sub)+'.txt', delimiter=',')
        # Read downstream hydrograph
        downhydro = numpy.loadtxt(outpath + 'downhydro_' + str(order)
        + '_' + str(sub)+'.txt', delimiter=',')
        # Artificially elongate the streams (*1.2) and there place
        # the downstream b.c.
        # This way the b.c.  will have less effect on the actual end of the reach.
        L_long=length*1.2
        Nx_long = int((L_long/dx)+1)
        init = numpy.array([base for i in range(0,Nx_long)])

        # Check if a reach drains into the considered reach.  This is done by checking
        # if the reach appears in column 2 and 3 of coupled_reaches
        check_lateral = numpy.where(numpy.logical_and(coupled_reaches[:,2]==order,
        coupled_reaches[:,3]==sub))[0]

        if len(check_lateral)==0:
            print 'reach ' + str(order) + '_' + str(sub) + ' has no sub-reaches'
            laterals=[]
        else:
            lateral=[]
            for i in check_lateral:
                # Append the cellnumber where the lateral flow should be added
                connect = numpy.load(outpath + 'cellnumber_' + str(coupled_reaches[i,0])
                + str(coupled_reaches[i,1]) + str(order) + str(sub) + '.npy')
                lateral.append(int(connect))

                # Append the lateral flow originating from the routing through
                # the channel -> Water_routed
                lat = numpy.loadtxt(outpath + 'lateral_' + str(coupled_reaches[i,0])
                + str(coupled_reaches[i,1]) + str(order) + str(sub)+'.txt', delimiter=',')
                for j in range(len(lat)):
                    lateral.append(lat[j])

                # Append the lateral flow originating from the runoff produced in
                # the cells (TotR as stored in TimeArrayPit) -> Water_system.
                # Determine location of outlets/pits of the reach that drains
                # into the regarded reach.
```

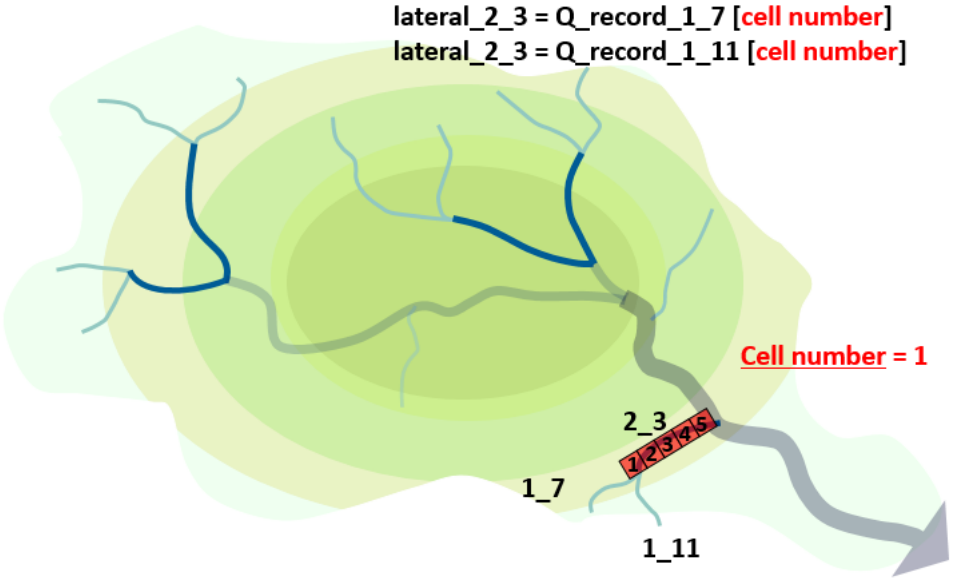
```
751                 rowIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]
                    ==float(order), outIDreach[:,1]==float(1)))[0][0]][2]
752                 colIDout = outIDreach[numpy.where(numpy.logical_and(outIDreach[:,0]
                    ==float(order), outIDreach[:,1]==float(1)))[0][0]][3]
753                 # Find where in pitID (= which row) the regarded pit is located
754                 indexOUT = numpy.where(numpy.logical_and(pitID[:,0]==rowIDout,
                    pitID[:,1]==colIDout))[0][0]
755                 latTotR = TimeArrayPit[1:,indexOUT+1]
756                 # If dt<24:  divide every timestep by 24/dt
757                 if dt < 24:
758                     latTotR_smalldt = []
759                     for i in range(len(latTotR)-1):
760                         t_lat = (latTotR[i+1]-latTotR[i])/(24/dt)
761                         for j in range(1,(24/dt)+1):
762                             latTotR_smalldt.append(latTotR[i]+t_lat*j)
763                     latTotR = latTotR_smalldt
764                 for k in range(len(latTotR)):
765                     lateral.append(latTotR[k])

767             # Every row in laterals covers the lateral hydrograph of one sub-reach
768             # that drains into the considered reach.  The first column of laterals
769             # is the cellnumber where the lateral flow enters the considered reach.
770             laterals=numpy.reshape(lateral, (len(check_lateral),len(lat)+1))
771             # Find whether there are multiple reaches flowing into the same cell of
772             # the next reach.  In that case these flows should be summed.
773             duplicates = [item for item, count in Counter(laterals[:,0]).iteritems()
                if count > 1]
774             # Find which rows in laterals have the same connect cell
775             for items in duplicates:
776                 index = numpy.argwhere(laterals[:,0]==items)
777                 # Add the time series of these lateral flows to 'temp'
778                 temp=[]
779                 for same in range(len(index)):
780                     temp.append(laterals[index[same][0],:])
781                 # Create empty array
782                 summed=numpy.zeros([len(temp[0]),0]).tolist()
783                 # Fill the list with zeros
784                 for j in range(1,len(temp[0])):
785                     summed[j]=0.
786                 # Add as first row the connectID cellnumber
787                 summed[0]=temp[0][0]
788                 # Add the summed time series to 'summed'
789                 for i in range(len(temp)):
790                     for j in range(1,len(temp[0])):
791                         summed[j]+=temp[i][j]
792                 summed=numpy.reshape(summed, (1,len(summed)))
793                 # Remove the individual time series of those that were just summed
794                 laterals = numpy.delete(laterals, index.tolist(), axis=0)
795                 # Add the summed time series
796                 laterals = numpy.concatenate([laterals,summed])
797                 # Update 'check_lateral'
798                 check_lateral = laterals[:,0]

800         # Prepare the distributed lateral inflow of the main stream.
801         # This comprises the accumulated runoff produced in the sub-catchment
802         # of the main outlet only, i.e.  the sub-catchments of all other reaches
```

```
803         # are substracted from this.  If not added laterally this volume of water
804         # would not be accounted for.
805         if order==strordmax:
806             lat_distr_maxorder = downhydro/len(reachIDs)
807         else:
808             lat_distr_maxorder = downhydro*0.

810         #### Apply cD-equation to the reach ####
811         Q_record = cD(uphydro, downhydro, init, laterals, check_lateral,
            lat_distr_maxorder, length, dx, dt, c, D)
812         numpy.savetxt(outpath + 'Q_record_' + str(order) + '_' + str(sub)
            + '.txt', Q_record, delimiter=',')

814         # The hydrograph of the last cell becomes the lateral flow for the reach
            # it drains into
815         if len(sub_downstream)>0:
816             lateral=[]
817             for i in range(len(Q_record)):
818                 lateral.append(Q_record[i][-1])
819             numpy.savetxt(outpath + 'lateral_' + str(order) + str(sub) + str(order_drain)
             + str(sub_drain)+'.txt', lateral, delimiter=',')

821         # Visualisation of the results?
822         if visFLAG==1:
823             # 3 options to add values to 'Qall' depending on relation dx:SpaceRes
824             # In the end the length of 'Qall' should be the same for the 3 options
825             # in order to visualize the results on a map with its
                # original spatial resolution
826             if dx == SpaceRes:
827                 Qall=numpy.concatenate([Qall,Q_record[:,1:]],axis=1)
828             else:
829                 Qtemp=[]
830                 if dx < SpaceRes:
831                     for times in range(len(Q_record)):
832                         Q_record_temp = Q_record[times,1:]
833                         # Take mean of every (SpaceRes/dx) cells
834                         means = numpy.mean(Q_record_temp.reshape(-1,int(SpaceRes/dx)),
                            axis=1)
835                         for cell in range(len(means)):
836                             Qtemp.append(means[cell])
837                     # In case the reach comprises only 1 cell
838                     if len(Q_record[0,:])==6 and len(reachIDs)==6:
839                         Qtemp=numpy.reshape(Qtemp, (len(Q_record),1))
840                     # In other cases
841                     else:
842                         Qtemp=numpy.reshape(Qtemp, (len(Q_record),len(reachIDs)))
843                     Qall=numpy.concatenate([Qall,Qtemp],axis=1)

845                 if dx > SpaceRes:
846                     # In case the reach comprises only 1 cell
847                     if len(Q_record[0,:])==2 and len(reachIDs)==6:
848                         Qtemp=numpy.reshape(Q_record[:,1], (len(Q_record),1))
849                         Qall=numpy.concatenate([Qall,Qtemp],axis=1)

851                     # In other cases duplicate each cell (dx/SpaceRes) times
852                     else:
```

```
853                             for times in range(len(Q_record)):
854                                 Q_record_temp = Q_record[times,1:]
855                                 # Repeat the value of a cell dx/SpaceRes times
856                                 rep = numpy.repeat(Q_record_temp, int(dx/SpaceRes))
857                                 # Remove at the beginning of the reach the number
858                                 # of cells that there are too many
859                                 if len(rep)>len(reachIDs):
860                                     rep = rep[(len(rep)-len(reachIDs)):]
861                                 # Repeat at the beginning of the reach the first value
862                                 # len(SubreachID)-len(rep)-times (pragmatic solution)
863                                 if len(rep)<len(reachIDs):
864                                     for i in range(len(reachIDs)-len(rep)):
865                                         rep=numpy.insert(rep, 0, rep[0])
866                                 # Add all values contained in rep to Qall
867                                 for cell in range(len(rep)):
868                                     Qtemp.append(rep[cell])
869                             Qtemp=numpy.reshape(Qtemp, (len(Q_record),len(reachIDs)))
870                             # Add all values contained in rep to Qall
871                             Qall=numpy.concatenate([Qall,Qtemp],axis=1)

873 if visFLAG==1:
874     numpy.savetxt(outpath + 'Qall.txt', Qall, delimiter=',')
```



(a) Run cD-equation for the reaches most upstream (smallest stream order). Continue this process in downstream direction.



(b) Retrieve 'lateral' from 'Q_record' for connecting downstream reaches.

Figure 6.13: 'Uphydro' and 'downhydro' were stored during the code run in section 6.8.5. These serve as input for the cD-equation of which the results are stored in 'Q_record' (Fig.(a)). The results of the outlet cell, as stored in 'Q_record', serves as lateral inflow of the connecting reach downstream (Fig.(b)).

6.10 Visualisation of the results

The results of the cD-equation are visualised using again the original PCRaster environment. For every time-step a map of the stream network will be created providing the output of the cD-equation for each cell. For this purpose the matrix 'networkID' is created in which the locations of all the cells of the whole stream network are stored.

```
407 # Put the locations of all cells of the network in table 'networkID'.
408 # Starting with the smallest stream order and per reach in downstream direction.
409 # This way the order will comply with the order as the values are stored in 'Q_record'
410 # after applying the cD-equation.
411 networkID = []
412 for order in range(strordmin,strordmax+1):
413     # Number of sub-reaches of the order (by counting the number of rows that have
414     # the same order)
415     Nsubs_this=inIDreach[:,0].tolist().count(float(order))
416     # Consider all sub-reaches that flow into this orders reach)
417     for sub in range(1,Nsubs_this+1):
418         # Read the table in which row and column for each cell are stored
419         # (column '1' and '2')
420         SubreachID = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order) +
          '_' + str(sub) + '.txt', delimiter=',')
421         # Add row and column (= location of cell) to 'networkID'
422             # Special case when reach comprises only 1 cell
423         if numpy.size(SubreachID)==6:
424             networkID.append(SubreachID[1])
425             networkID.append(SubreachID[2])
426             # All other cases
427         else:
428             for ID in range(len(SubreachID)):
429                 networkID.append(SubreachID[ID,1])
430                 networkID.append(SubreachID[ID,2])

432 # Reshape the whole list into an matrix of 2 columns.  Column '0' = row,
433 # column '1' = column
434 networkID=numpy.reshape(networkID, (len(networkID)/2,2))
435 numpy.savetxt(outpath + 'networkID.txt', networkID, delimiter=',')
```

The last step in the visualisation consists of combining 'networkID' and 'Qall' and create for each time step a map.

```
879 if visFLAG==1:
880     # Use 'Qall' to visualise the results by creating maps with original
881     # spatial resolution.  Create a map that only contains the stream network
        # and convert it to a numpy array (float).
882     network_solely = pcr.ifthen(network,clone)
883     pcr.report(network_solely, outpath + 'network_solely.map')
884     output_array = pcr.pcr2numpy(network_solely, MV).astype(numpy.float)
885     # Set initials for dynamically naming the files
886     Q=0
887     count=0
888     pcrcounter=0.
889     endcount=dt_visualize/dt
890     # For each timestep create a map
891     for times in range(len(Qall)):
892         count+=1
```

```
893           Q+=Qall[times]
894           if count == endcount:
895                count=0
896                Qall_vis=Q/(dt_visualize/dt)
897                Q=0
898                # Consider all cells of the stream network and put the accompanying
899                # value of 'Qall' at the right location in the numpy array.  Then convert
900                # the resulting array back to a PCRaster map
901                for ID in range(len(networkID)):
902                     output_array[int(networkID[ID,0]),int(networkID[ID,1])]= Qall_vis[ID]
903                result = pcr.numpy2pcr(Scalar, output_array, output_array[0,0])
904                pcrcounter+=1
905                pcrstr = '%011.3f' %(pcrcounter/1000)
906                pcr.report(result,outpath + 'Q' + pcrstr)
```

6.11 Time series at observation stations

For calibration purposes time series of the simulated results at the observation stations are provided at a daily base (r.932–947) in the original spatial resolution (r.949–997). Discharge observations and simulated discharge using the cD-routing procedure are plotted, as well as discharge simulated with the current simple routing scheme are plotted for comparison (r.1012–1050). Nash-Sutcliffe values are displayed as an indication of model performance (r.1052–1064).

```
912  # Time series of specified locations, e.g.  streamflow locations
913  for loc in range(len(stations_loc)-1):  # -1 to discard the outlet of the total area
914       # Determine in which reach the station is located
915       order = int(stations_loc[loc,2])
916       sub = int(stations_loc[loc,3])
917       index = int(stations_loc[loc,4])
918       index = int(index * (grid_res/dx))

920       # Determine nameID of the station
921       name = int(names[int(stations_loc[loc,0]),int(stations_loc[loc,1])])
922       if name == 999:
923            name = 'outlet'
924       name = str(name)

926       # Read the locations of the cells belonging to the reach that is considered
927       reachIDs = numpy.loadtxt(outpath + 'SubReachArrayID_' + str(order) +
          '_' + str(sub) + '.txt', delimiter=',')

929       #### Handling Q_station files when dt != 24 ####
930       # Read simulated Q of the reach
931       Q_station = numpy.loadtxt(outpath + 'Q_record_' + str(order) + '_' +
          str(sub) + '.txt', delimiter=',')
932       if dt<24:
933            Qstation_dt = []
934            Q=0
935            count=0
936            endcount=24/dt
937            # For each timestep create a map
938            for times in range(len(Q_station)):
939                 count+=1
940                 Q+=Q_station[times]
```

```
941             if count == endcount:
942                 count=0
943                 Qstation_graph=Q/(24/dt)
944                 Q=0
945                 Qstation_dt.append(Qstation_graph)
946         Qstation_dt=numpy.reshape(Qstation_dt, (len(Q_station)/(24/dt),len(Q_station[0])))
947         Q_station = Qstation_dt

949     #### Handling Q_station files when dx != SpaceRes ####
950     Qall=numpy.empty([len(downhydro_maxorder),0])
951     if dx != SpaceRes:
952         Qtemp=[]
953         if dx < SpaceRes:
954             for times in range(len(Q_station)):
955                 Q_record_temp = Q_station[times,1:]
956                 # Take mean of every (SpaceRes/dx) cells
957                 means = numpy.mean(Q_record_temp.reshape(-1,int(SpaceRes/dx)), axis=1)
958                 for cell in range(len(means)):
959                     Qtemp.append(means[cell])
960             # In case the reach comprises only 1 cell
961             if len(Q_station[0,:])==6 and len(reachIDs)==6:
962                 Qtemp=numpy.reshape(Qtemp, (len(Q_station),1))
963             # In other cases
964             else:
965                 Qtemp=numpy.reshape(Qtemp, (len(Q_record),len(reachIDs)))
966             Qall=numpy.concatenate([Qall,Qtemp],axis=1)
967             Q_station=Qall

969         if dx > SpaceRes:
970             # In case the reach comprises only 1 cell
971             if len(Q_station[0,:])==2 and len(reachIDs)==6:
972                 Qtemp=numpy.reshape(Q_station[:,1], (len(Q_station),1))
973                 Qall=numpy.concatenate([Qall,Qtemp],axis=1)
974                 Q_station=Qall

976             # In other cases duplicate each cell (dx/SpaceRes) times
977             else:
978                 for times in range(len(Q_station)):
979                     Q_record_temp = Q_station[times,1:]
980                     # Repeat the value of a cell dx/SpaceRes times
981                     rep = numpy.repeat(Q_record_temp, int(dx/SpaceRes))
982                     # Remove at the beginning of the reach the number of cells
983                     # that there are too many
984                     if len(rep)>len(reachIDs):
985                         rep = rep[(len(rep)-len(reachIDs)):]
986                     # Repeat at the beginning of the reach the first value
987                     # len(SubreachID)-len(rep)-times (pragmatic solution)
988                     if len(rep)<len(reachIDs):
989                         for i in range(len(reachIDs)-len(rep)):
990                             rep=numpy.insert(rep, 0, rep[0])
991                     # Add all values contained in rep to Qall
992                     for cell in range(len(rep)):
993                         Qtemp.append(rep[cell])
994                 Qtemp=numpy.reshape(Qtemp, (len(Q_record),len(reachIDs)))
995                 # Add all values contained in rep to Qall
996                 Qall=numpy.concatenate([Qall,Qtemp],axis=1)
```

```
997                 Q_station=Qall

999     # Read observed Q and Q simulated using the old routing procedure
1000     Q_obs_sim = PlottingObsSim.ObsSim(name,inpath,sy,sm,sd)

1002     # Create array containing dates.
1003     date = numpy.arange(datetime.date(sy,sm,sd), datetime.date(ey,em,ed),
        datetime.timedelta(days=1)).astype(datetime.date)
1004     # Date formatter
1005     years = mdates.YearLocator()
1006     months = mdates.MonthLocator()
1007     if len(date)>365:
1008         Fmt = mdates.DateFormatter('%Y')
1009     if len(date)<366:
1010         Fmt = mdates.DateFormatter('%m-%Y')

1012     # Create plots
1013     fig, ax = pyplot.subplots()
1014     ax.plot(date, Q_obs_sim[0][Q_obs_sim[1]:(Q_obs_sim[1]+len(date))],
        color='grey', label='Observed')
1015     ax.plot(date, Q_obs_sim[2][Q_obs_sim[3]:(Q_obs_sim[1]+len(date))],
        color='dodgerblue', label='Simple routing')
1016     if len(Q_station)>len(date):
1017         diff = abs(len(Q_station)-len(date))
1018         ax.plot(date, Q_station[diff:,index],color='darkorange', label='cD-routing')
1019     else:
1020         ax.plot(date, Q_station[:,index],color='darkorange', label='cD-routing')

1022     # Add label and legend
1023     pyplot.ylabel('Discharge [m$^3$ s$^{-1}$]')
1024     pyplot.legend(fontsize=9,frameon=False)

1026     # format the ticks
1027     if len(date)>365:
1028         ax.xaxis.set_major_locator(years)
1029         ax.xaxis.set_major_formatter(Fmt)
1030         ax.xaxis.set_minor_locator(months)

1032     if len(date)>365:
1033         ax.xaxis.set_major_locator(months)
1034         ax.xaxis.set_major_formatter(Fmt)

1036     datemin = datetime.date(date.min().year, 1, 1)
1037     datemax = datetime.date(date.max().year + 1, 1, 1)
1038     ax.set_xlim(datemin, datemax)

1040     # rotates and right aligns the x labels, and moves the bottom of the
1041     # axes up to make room for them
1042     for ax in fig.get_axes():
1043         if ax.is_last_row():
1044             for label in ax.get_xticklabels():
1045                 label.set_ha('right')
1046                 label.set_rotation(30.)
1047         else:
1048             for label in ax.get_xticklabels():
1049                 label.set_visible(False)
```

```
1050              ax.set_xlabel('')

1052      # Prepare to calculate Nash-Sutcliffe value
1053      old = pd.DataFrame('obs':Q_obs_sim[0][Q_obs_sim[1]:(Q_obs_sim[1]+len(date))],
          'sim_old':Q_obs_sim[2][Q_obs_sim[3]:(Q_obs_sim[1]+len(date))])
1054      if len(Q_station)>len(date):
1055          diff = abs(len(Q_station)-len(date))
1056          new = pd.DataFrame('obs':Q_obs_sim[0][Q_obs_sim[1]:(Q_obs_sim[1]+len(date))],
              'sim_new':Q_station[diff:,index])
1057      else:
1058          new = pd.DataFrame('obs':Q_obs_sim[0][Q_obs_sim[1]:(Q_obs_sim[1]+len(date))],
              'sim_new':Q_station[:,index])
1059      # Calculate NS values
1060      optFunctions.NS(old.obs,old.sim_old)
1061      optFunctions.NS(new.obs,new.sim_new)
1062      # Plot NS values in graphs
1063      pyplot.text(0.25,0.92,"NS_cD = " + "%.2f" % optFunctions.NS(new.obs,new.sim_new),
          transform=ax.transAxes,fontsize=9)
1064      pyplot.text(0.25,0.85,"NS_simple = " + "%.2f" % optFunctions.NS(old.obs,old.sim_old),
          transform=ax.transAxes,fontsize=9)

1066      # Save the results
1067      pyplot.savefig(outpath + 'Q_loc_'+name+'_c' + str(c) + '_D' + str(D) +
          '_dt' + str(dt) + '_dx' + str(dx) + '.pdf')
1068      numpy.savetxt(outpath + 'Q_loc_'+name+'_c' + str(c) + '_D' + str(D) +
          '_dt' + str(dt) + '_dx' + str(dx) +'.txt', Q_station[1:,index], delimiter=',')
```

6.12 Calibration

Apart from SPHY parameters that need to be calibrated (i.e. they are influencing simulated specific runoff as used by the routing module), there are two parameters in the routing module that need to be calibrated: wave celerity (c) and diffusion coefficient (D). Both can also be calculated from the linearised cD-equation (see Eq.6.2 and 6.3). However, this may not always yield good results as was explained in section 6.6, e.g. erratic behaviour of discharge or too steep recession curves, resulting in low Nash-Sutcliffe values (Nash and Sutcliffe, 1970). Therefore for this application example both values have been calibrated manually. To evaluate the performance of the routing procedure in the SPHY model, the Nash-Sutcliffe efficience (NS) of the discharge is computed as a measure of goodness of fit.

Values of these calibration parameters depend on the period of time that is simulated, as well as the selected spatial and temporal resolution. Changes made in one of these three variables require recalibration of the parameters.

Currently the parameters c and D are spatially constant for the entire basin, while spatially dependent values for these parameters may yield different and better results. However, this requires calibration of these parameters for each reach in the stream network or at locations in which the characteristics of the stream change substantial.

To test several combinations of parameters before running the whole routing module, the user can opt to run the script '*cD_test.py*' (see Appendix C). For this the user needs to specify ranges for the parameters c, D, dt and dx and provide an estimation of the expected maximum and minimum discharge. The code will provide a log-file with the parameter values, Courant number, Peclet number, maximum and minimum simulated discharge, total discharge upstream and downstream and it will raise a warning if large oscillations occurred. The latter is an indication that the parameter combination should be changed.

6.13 Adjustments made in SPHY script

This whole routing procedure is applied at the end of the SPHY model when the total specific runoff for each cell has been determined. The following code has been added to the SPHY programming code:

```
1303 #-cD-routing module
1304 if self.curdate == self.enddate:
1305     if self.CDRoutFLAG == 1:

1307         #-spatial step (in km)
1308         try:
1309             self.dx = config.getfloat('CDROUTING', 'dx')
1310         except:
1311             self.dx = self.SpaceRes

1318         # Check the ratio between dx and SpaceRes
1314         if self.dx > self.SpaceRes:
1315             test = self.dx/self.SpaceRes
1316             # Check if the float number is a whole number
1317             if test.is_integer():
1318                 print ''
1319             else:
1320                 sys.exit('ERROR: dx/SpaceRes is not an integer:  Change dx')
1321         if self.dx < self.SpaceRes:
1322             test = self.SpaceRes/self.dx
1323             if test.is_integer():
1324                 print ''
1325             else:
1326                 sys.exit('ERROR: SpaceRes/dx is not an integer:  Change dx')

1328         #-time step (in hours)
1329         self.dt = config.getint('CDROUTING', 'dt')
1330         dt_options = [1,3,6,12,24]
1331         if self.dt not in dt_options:
1332             sys.exit('ERROR: chosen dt does not correspond to the options
                provided:  change dt to 1, 3, 6, 12 or 24.')

1334         #-visualization time step (in hours)
1335         self.dt_visualize = config.getint('CDROUTING', 'dt_visualize')
1336         dt_options = [1,3,6,12,24]
1337         if self.dt_visualize not in dt_options:
1338             sys.exit('ERROR: chosen dt_visualize does not correspond to the options
                provided:  change dt to 1, 3, 6, 12 or 24.')

1340         #-check if dt_visualize is larger than dt
1341         if self.dt_visualize < self.dt:
1342             sys.exit('ERROR: dt_visualize is smaller than dt:  choose a value for
                dt_visualize equal to or larger than dt')

1344         #-wave celerity and diffusion coefficient
1345         try:
1346             self.c = config.getfloat('CDROUTING', 'c')
1347             self.D = config.getfloat('CDROUTING', 'D')
1348         except:
1349             self.width = config.getfloat('CDROUTING','width')
1350             self.depth = config.getfloat('CDROUTING','depth')
```

```
1351              self.roughness = config.getfloat('CDROUTING','roughness')
1352              self.slope = config.getfloat('CDROUTING','slope')
1353              self.c = self.cD.calcCD(self,self.width,self.depth,
                  self.roughness,self.slope)[0]
1354              self.D = self.cD.calcCD(self,self.width,self.depth,
                  self.roughness,self.slope)[1]
1355              print 'c and D are calculated'
1356          if self.c*(self.dt*60*60)/(self.dx*1000) > 1:
1357              print 'WARNING: Courant number is larger than 1, this could lead to
                  numerical instability:  Decrease time step, or increase spatial step'

1359          self.CDrouting.CDrouting(self,self.dx,self.dt,self.SpaceRes,
              self.dt_visualize,self.c,self.D)
```

7. Results and Discussion

7.1 Model output and analysis

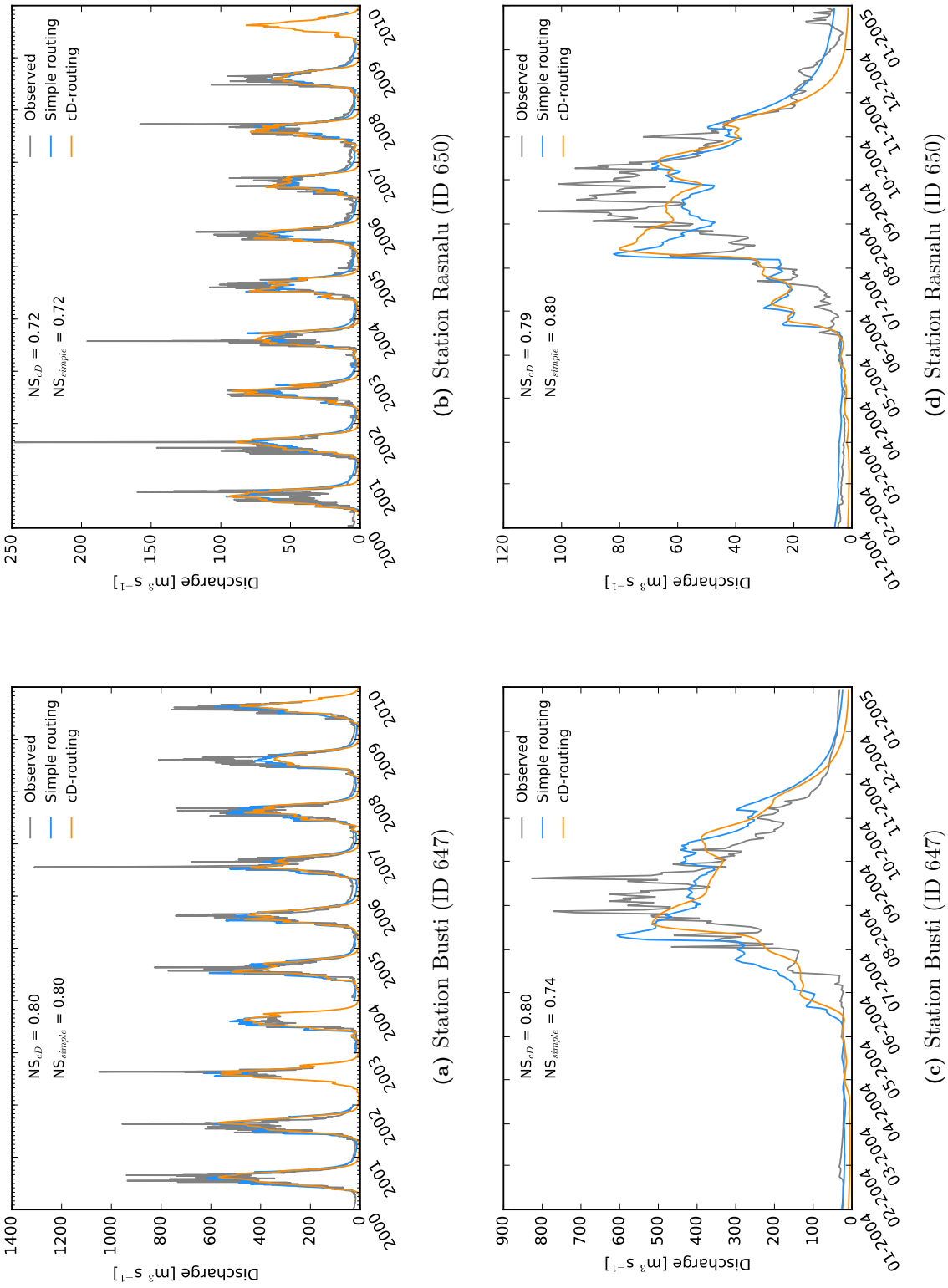
Figures 7.1a and 7.1b show daily observed and SPHY simulated discharge using the current simple routing scheme and using the newly implemented cD-routing procedure. The flow hydrographs are shown for 10 consecutive years at the streamflow stations Busti (ID 647) and Rasnalu (ID 650), that both are located in the Tamakoshi river basin. Values of c (= 0.08 km hr^{-1}) and D (= 1.0 km^2 hr^{-1}) were manually calibrated. However, these values make no sense physically as they are relatively low and as they are not comparable to values found in literature (Vakgroep Hydraulica en Afvoerhydrologie, Year unknown) and when calculated using equations 6.2 and 6.3. Despite this the model performs well given a NS value of 0.80 and 0.72 for stations Busti and Rasnalu respectively for both routing procedures. In general the average simulated discharge corresponds to the observed discharge. However, peak discharges are frequently underestimated, as well as the low flows during the months December – May.

During this application example the settings of the SPHY model, as have been provided from a previous study, have not been changed. Recalibration of the SPHY model could change the simulated specific runoff for each cell, which is used as input for the cD-routing module, could further improve the results of the cD-routing module.

Comparing the new cD-routing procedure to the current implemented simple routing scheme in SPHY it can be seen in figures 7.1a and 7.1b that the results of both simulations are comparable. Both yield the same NS value when considering the 10 years data series. The strong seasonality has strong influence on the resulting reasonably high NS value of 0.80. This however, does not necessarily mean that the model performs well. In general the simple routing scheme simulates higher peaks than using the cD-routing. Low flows during the months December – May are better simulated by the simple routing. Zooming into the year 2004 it is demonstrated that the observed streamflow is more erratic than both simulated runs (see Fig. 7.1c and 7.1d).

Figure 7.2 demonstrates the results of daily observed and SPHY simulated discharge for the same locations after calculating the values of c (= 18.5 km hr^{-1}) and D (= 0.18 km^2 hr^{-1}) using equations 6.2 and 6.3. It can be seen that simulated discharge behaves erratic and the average discharge is too low, resulting in a bad model performance with NS values of -0.13 and -0.31 for stations Busti and Rasnalu respectively. The erratic behaviour is most probably due to the high wave celerity causing a wave to pass through the channels fast. This will result in a steep rising limb and recession limb of the hydrograph.

An example of the visualisation of the simulation results is shown in figure 7.3. It displays the streamflow for each cell of the stream network at a certain timestep. By displaying a number maps of consecutive time-steps after each other a movie can be created in which the propagation of a wave can be visualised as it travels through the stream network.



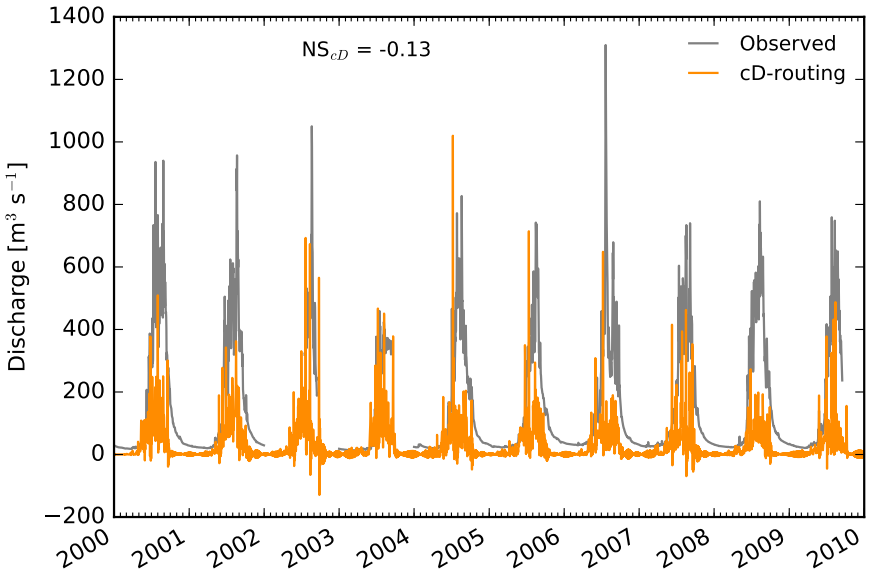
(a) Station Busti (ID 647)

(b) Station Rasnalu (ID 650)

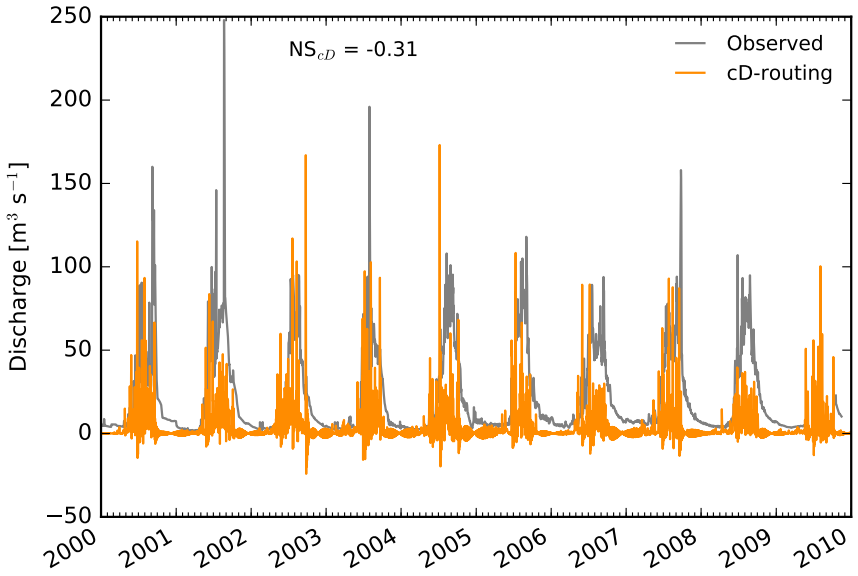
(c) Station Busti (ID 647)

(d) Station Rasnalu (ID 650)

Figure 7.1: Daily observed discharge, SPHY simulated discharge using the simple routing procedure and SPHY simulated discharge using the cD-routing procedure for the streamflow stations Busti (ID 647) and Rasnalu (ID 650) for 10 years (a & b) and for the year 2004 (c & d). Values of c and D were calibrated based on the 10 year data series.



(a) Station Busti (ID 647)



(b) Station Rasnalu (ID 650)

Figure 7.2: Daily observed and SPHY simulated discharge using the cD-routing procedure for the streamflow stations Busti (ID 647) and Rasnalu (ID 650). Values of c and D were calculated using equations 6.2 and 6.3.

Table 7.1: Overview of the results of the model performance.

(a) Overview of the results of the model performance with changing temporal resolution. The parameters c, D and dx are kept constant at 0.08 [km hr^{-1}], 0.1 [km^2 hr^{-1}] and 0.250 km respectively.

	Station Busti							Station Rasnalu						
	2000 – 2010			**Year 2004**				**2000 – 2010**			**Year 2004**			
	Simple routing	cD-routing		Simple routing	cD-routing			Simple routing	cD-routing		Simple routing	cD-routing		
dt [hr]	24	12	24	24	1	12	24	24	12	24	24	1	12	24
NS [–]	0.80	0.74	0.80	0.74	-42	0.70	0.80	0.72	0.72	0.72	0.80	0.79	0.79	0.79
Computational time [min]	–	7	4	–	8	1	0.5	–	7	4	–	8	1	0.5

(b) Overview of the results of the model performance with changing spatial resolution. The parameters c, D and dt are kept constant at 0.08 [km hr^{-1}], 0.1 [km^2 hr^{-1}] and 24 hr respectively.

	Station Busti							Station Rasnalu						
	2000 – 2010			**Year 2004**				**2000 – 2010**			**Year 2004**			
	Simple routing	cD-routing		Simple routing	cD-routing			Simple routing	cD-routing		Simple routing	cD-routing		
dx [km]	0.25	0.25	0.50	0.25	0.05	0.25	0.50	0.25	0.25	0.50	0.25	0.05	0.25	0.50
NS [–]	0.80	0.80	0.79	0.74	0.47	0.80	0.79	0.72	0.72	0.71	0.80	0.79	0.79	0.78
Computational time [min]	–	4	2	–	13	0.5	0.25	–	4	2	–	13	0.5	0.25

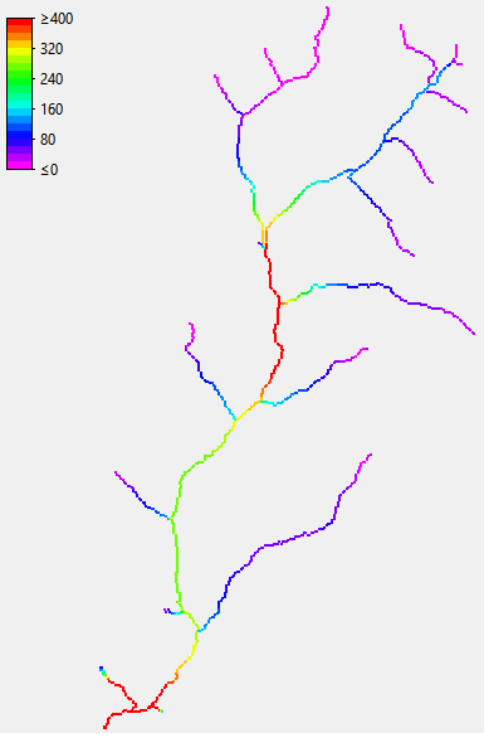


Figure 7.3: Example of the visualisation of the simulation results using the cD-routing procedure. Red colours indicate high streamflow (in $m^3\ s^{-1}$) and purple colours low streamflow.

7.2 Sensitivity analysis

A sensitivity analysis has been performed to assess the sensitivity of the routing module to changes in the parameters c and D, as well as changes in the temporal and spatial resolution, i.e. dt and dx respectively.

7.2.1 Wave celerity

Figures 7.4a and 7.4b display the results of the sensitivity analysis of parameter c for a period of 10 years for both stations respectively. In orange the default simulation with c = 0.08 km hr^{-1} is depicted. The other lines demonstrate the results of decreasing and increasing the value of c with 25% and 50%. A higher value of c results in lower simulated discharges and a more erratic hydrograph. It was expected, supported by performed artificial examples, that discharge peaks would be higher with higher values of c, because the wave would arrive at the location sooner with less time for the development of diffusive effects. A small shift can be seen in the arrival of the peak with changing wave celerity. This effect might become more clearly visible when displaying over a shorter period of time of for instance a week. The peaks simulated with higher wave celerity are sharper as expected, because the wave takes less time to pass the station. Moreover, it becomes clear that mass is not preserved and this seems to be the largest issue here. No cause could be found yet to explain this strange and unexpected behaviour.

Zooming in to the year 2004 (Fig. 7.4c and 7.4d) it becomes visible for station Busti that during the recession limb the lines start to differ more than during the rising limb. This implies that at the arrival of peak discharges the routing module is less sensitive to changes in c than during the recession limb. This makes it more difficult to calibrate c such that it simulates the peaks well at this station. Station Rasnalu shows more sensitivity at the rising limb compared to the other station. This might be attributable to the size of the area upstream of the location, which is much smaller for Rasnalu than for Busti. Further analysis is required to assess this assumption.

7.2.2 Diffusion coefficient

Figures 7.5a and 7.5b display the results of the sensitivity analysis of parameter D for a period of 10 years for both stations respectively. In orange the default simulation with D = 0.1 km^2 hr^{-1} is depicted. The other lines demonstrate the results of decreasing and increasing the value of D with 25% and 50%. Higher values of D result in smoother hydrographs and in higher simulated discharges. The latter is opposite to what is expected, namely higher values of D leading to more attenuated waves and therefore lower peaks. This might be a result of the numerical scale chosen too large with respect to the system time scale. Further analysis is needed to assess this. Again it is visible that mass is not preserved and this needs further exploration to be able to explain and solve this problem. From a pragmatic point of view the value of D can be seen as a parameter that can be used to calibrate the routing module.

Zooming in to the year 2004 (Fig. 7.5c and 7.5d) it can be seen that with higher D the total increase and decrease of discharge during peaks is higher. Peaks are therefore simulated higher, but the recession after is larger as well. Similar as to the parameter c, discharge at station Busti is less sensitive to changes in the parameter D than at station Rasnalu.

Considering the period of 10 years from 2000 – 2010 as a reference it can be seen that the cD-routing module is more sensitive to the wave celerity compared to the diffusion coefficient.

7.2.3 Temporal resolution

Sensitivity to changes in the temporal resolution is displayed in figure 7.6. Dt is given in hours and is indicating the temporal resolution on which the calculation of the routing is performed (i.e. visualisation is performed on a daily base). In orange the default simulation at dt = 24 hr is shown. An hourly temporal resolution is not included in the analysis over the period 2000 – 2010 since it required more storage memory (RAM memory) than available.

The routing module appears to be very sensitive to the temporal resolution at station Busti (Fig. 7.6c), where a finer temporal resolution results in significantly higher simulated discharge. It is not fully understood why calculations performed on an hourly base lead to rather different results than coarser temporal resolutions. Therefore it would require recalibration of the parameters c and D when changing the temporal resolution. Due to time restrictions it was not possible to assess the results of such a recalibration. In addition, the higher sensitivity to the temporal resolution at station Busti compared to station Rasnalu might partly be attributed to the size of the catchment upstream as this is the main difference between the two stations. This needs further exploration. And again there is the issue of the mass that is not conserved with changing temporal resolution.

7.2.4 Spatial resolution

In figure 7.7 the sensitivity of the routing module to changes in the spatial resolution are shown. Dx is given in metres and the temporal resolution is kept constant at 24 hours. Values of c and D are kept constant as well at 0.08 km hr^{-1} and 0.1 km^2 hr^{-1} respectively. In orange the default simulation at dx = 250 m is depicted. A spatial resolution of 50 m is not included in the analysis over the period 2000 – 2010 since it required more storage memory than available.

A finer spatial resolution results in substantial lower simulated discharges at station Busti, while discharge simulated at station Rasnalu is insensitive such a change. And here as well mass is not preserved with changing spatial resolution. Simulating discharge at a larger spatial resolution for both stations does not significantly change the results. Again there is no explanation for this phenomenon yet and therefore needs further assessment. Again the size of the upstream catchment is a possible factor that could explain these results, or a mistake made in the programming code could be the explanation. Another difference between these stations is the stream order of the reach in which it is located. Station Busti is located in the main stream of the stream network, while station Rasnalu is located in a reach of a lower stream order. Only in the main stream the lateral inflow is added in a distributed way instead of point injection (see Sect. 6.5). Further analysis is needed to assess whether this interferes with the code for running the routing module on a smaller spatial resolution. In addition due to time restrictions it was not possible

Table 7.2: Computational time in minutes for a simulation period of 10 years on a daily base with the original spatial resolution (= 250m) for the two different routing procedures.

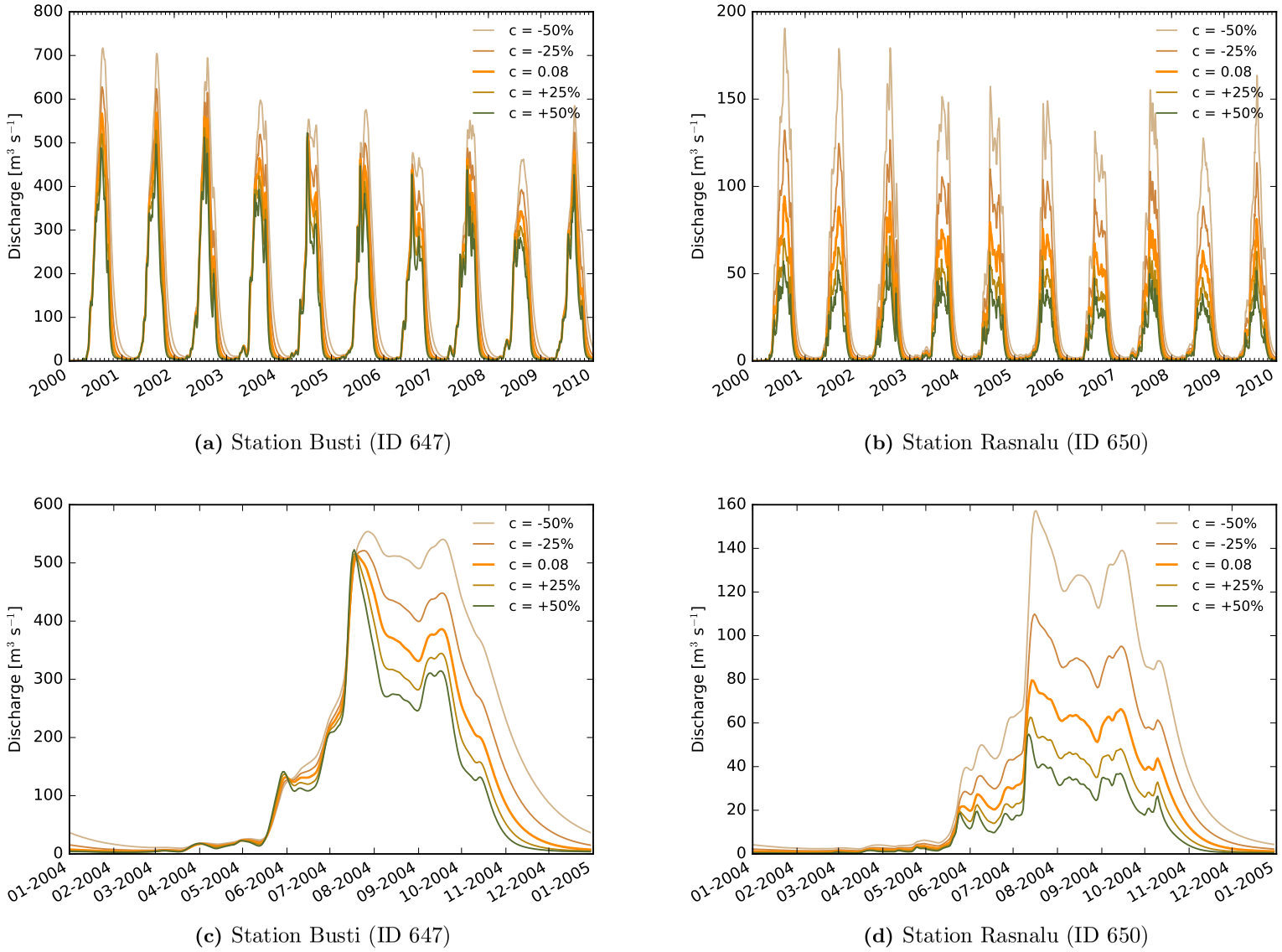
	Simple routing	**cD-routing**
Preprocessing	–	0.2
Calculating accumulated runoff	–	18
Routing without/with visualisation	–	4/7
Total	**≈ 20**	**22/25**

to assess the results of recalibration of the model with changing spatial resolutions.

7.3 Computational time

Preferably the computational time of the new cD-routing module does not exceed much the more simple current routing scheme. In table 7.1 the computational time for different scenarios is depicted. Note that these number indicate the time it takes to run only the routing part (i.e. applying the cD-equation) of the routing module (r.516 – r.1068) (without creating maps). This is the most relevant as this part will be used when calibrating the routing parameters. The computational time of all the preprocessing steps in which the stream network is defined and all the individual reaches are retrieved (r.27 – r.402) is 10 seconds. The time to compute and prepare the tables containing the accumulated runoff for all time-steps for several points (r.435 – r.503) depends on the period of time considered: for a period of 1 year this takes 1.5 minutes, while for a period of 10 years it takes 18 minutes. In table 7.2 an overview is provided of these computational times for a simulation period of 10 years on a daily base with a spatial resolution of 250m. Furthermore, the total computational time of the two routing procedures is provided.

In addition it should be noted that the routing module can be run independently of the SPHY model. Therefore the SPHY model should be run once to calculate and create the maps containing the total accumulated specific runoff for each cell. Subsequently the routing module can be run several times with different settings without having to run the whole SPHY model again.



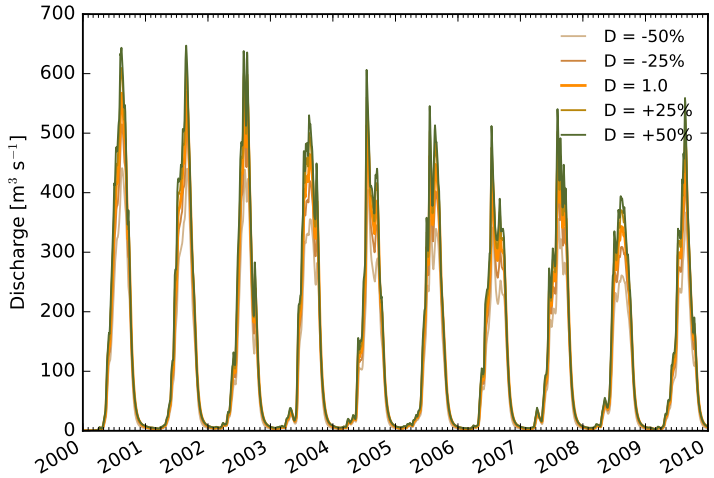
(a) Station Busti (ID 647)

(b) Station Rasnalu (ID 650)

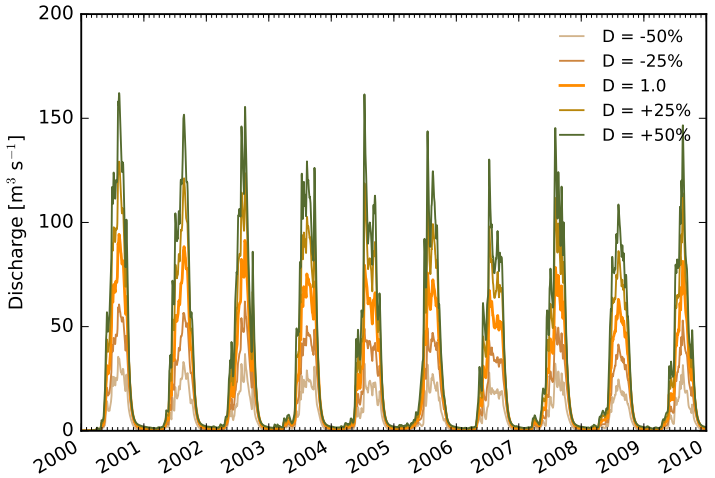
(c) Station Busti (ID 647)

(d) Station Rasnalu (ID 650)

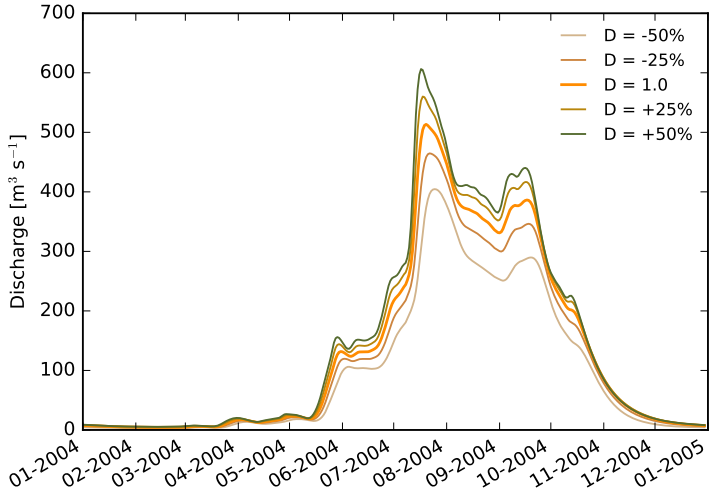
Figure 7.4: Sensitivity analysis of wave celerity for 10 consecutive years ((a) and (b)) and for the year 2004 ((c) and (d)). Values of c are given in km hr^{-1}. The diffusion coefficient is kept constant at 1.0 km^2 hr^{-1}.



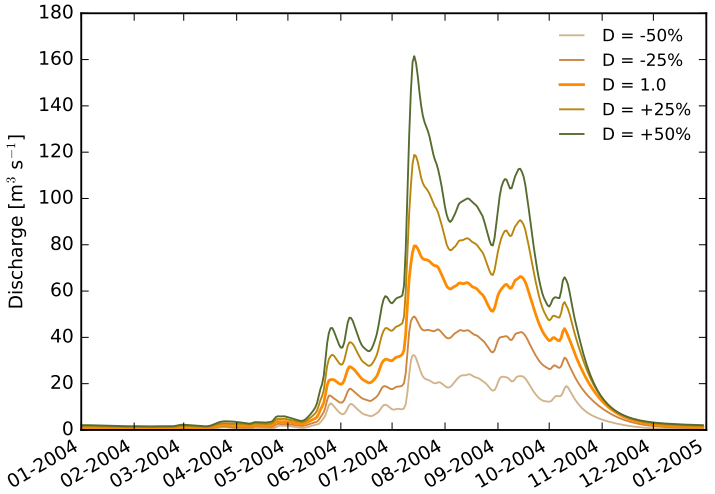
(a) Station Busti (ID 647)



(b) Station Rasnalu (ID 650)

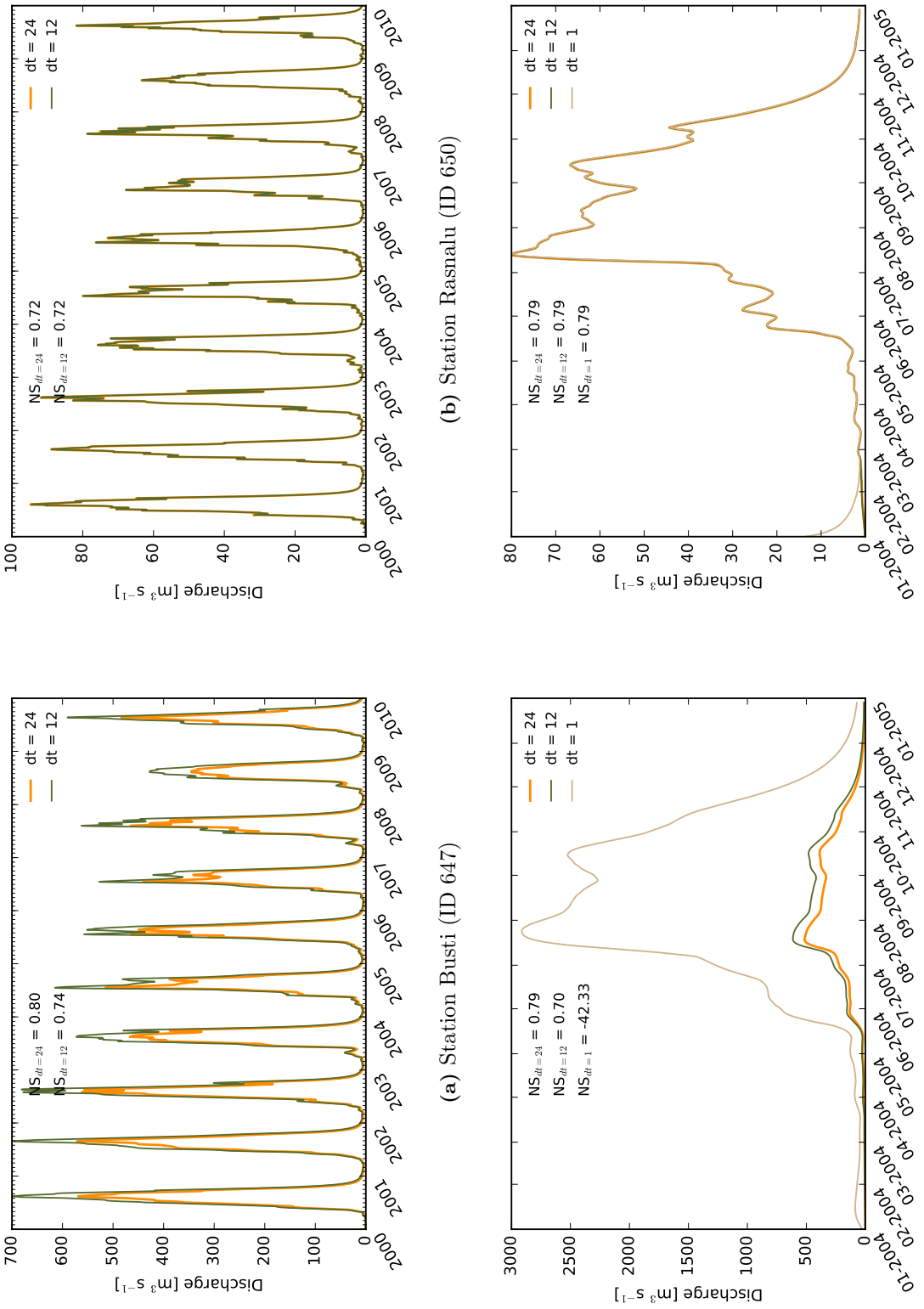


(c) Station Busti (ID 647)



(d) Station Rasnalu (ID 650)

Figure 7.5: Sensitivity analysis of the diffusion coefficient for 10 consecutive years ((a) and (b)) and for the year 2004 ((c) and (d)). Values of D are given in km^2 hr^{-1}. The wave celerity is kept constant at 0.08 km hr^{-1}.



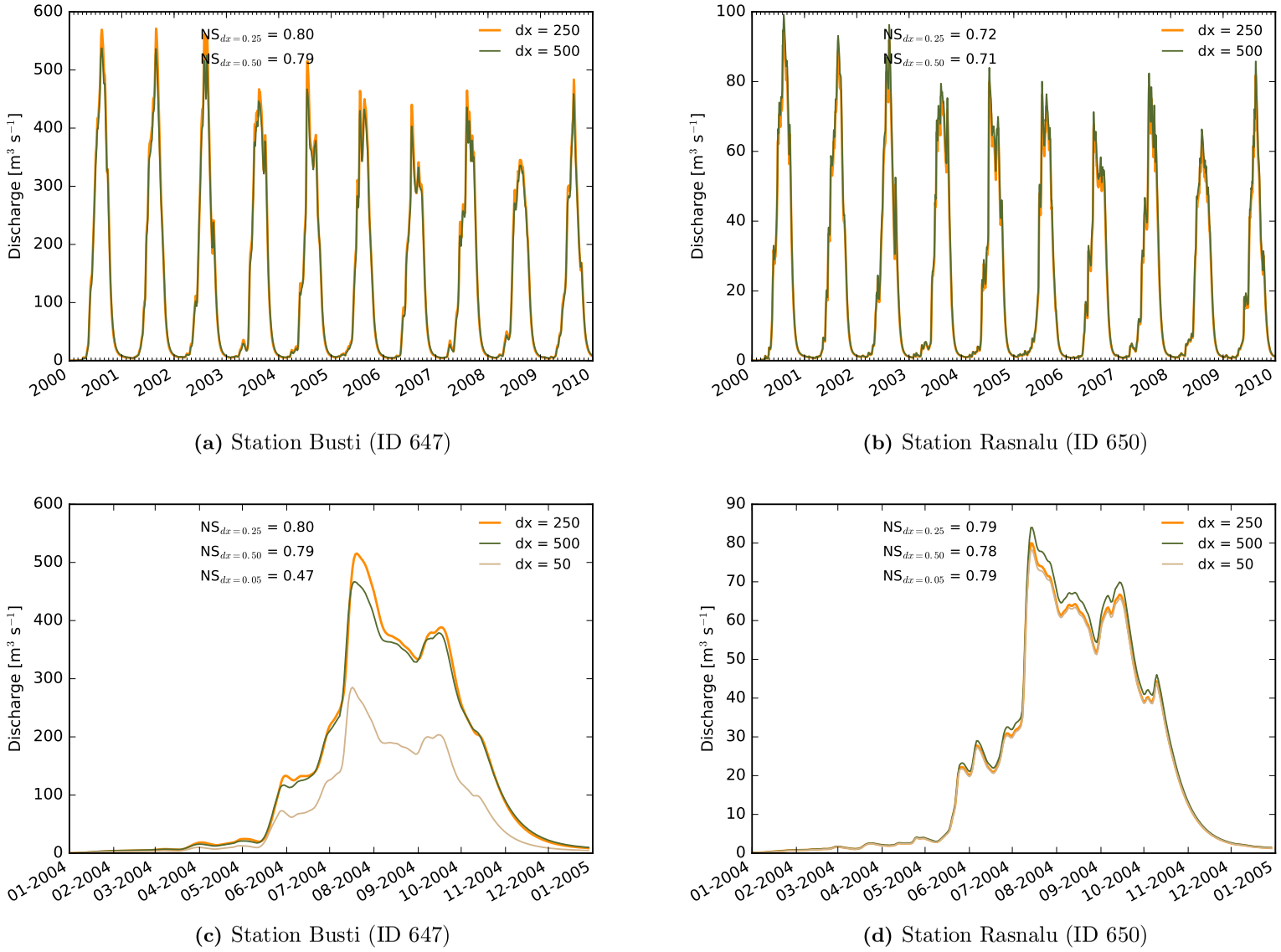
(a) Station Busti (ID 647)

(b) Station Rasnalu (ID 650)

(c) Station Busti (ID 647)

(d) Station Rasnalu (ID 650)

Figure 7.6: Sensitivity analysis of the temporal resolution for 10 consecutive years ((a) and (b)) and for the year 2004 ((c) and (d)). Dt is given in hours and spatial resolution is kept constant at 0.25 km. Values of c and D are kept constant as well at 0.08 km hr^{-1} and 0.1 km^2 hr^{-1} respectively.



(a) Station Busti (ID 647)

(b) Station Rasnalu (ID 650)

(c) Station Busti (ID 647)

(d) Station Rasnalu (ID 650)

Figure 7.7: Sensitivity analysis of the spatial resolution for 10 consecutive years ((a) and (b)) and for the year 2004 ((c) and (d)). Dx is given in metres and temporal resolution is kept constant at 24 hr. Values of c and D are kept constant as well at 0.08 km hr^{-1} and 0.1 km^2 hr^{-1} respectively.

8. Conclusion

In this internship research project alternative routing procedure options were explored with the aim to improve the current routing scheme and therefore streamflow simulations in the SPHY model. Given the fact that SPHY is regularly applied in data-scarce research areas, it is concluded that the convection-diffusion (cD) approach is the most suitable routing method. The cD-wave equations provide a solid approximation of the full dynamic wave equations as is balances between the accuracy and high data demanding dynamic wave model and the simplicity of the kinematic wave model. The cD-equation was numerically discretized using the Crank-Nicolson scheme.

The cD-routing module is set up outside the PCRaster environment. This provides the opportunity to run the routing module with a different temporal and spatial resolution than the simulation of the SPHY model itself. Furthermore, visualisation of the results can be done on a different time-step than the routing calculations. These settings need to be specified by the user in the accompanying SPHY configuration file.

For the implementation of the cD-routing procedure it was necessary to first define the stream network and to identify all individual reaches of the different stream orders. In addition, it was determined which reaches are connecting to each other, since the output of one stream is added as lateral inflow into the next. Furthermore, time series containing the accumulated runoff for specified points (e.g. inlets and outlets of reaches) were stored in tables. After determining the downstream hydrographs for all reaches by applying the cD-routing procedure to all individual reaches starting at the downstream point and proceeding in upstream direction, the main loop starts. In final part the actual routing is performed, for which the cD-equation was applied to all reaches starting upstream and proceeding in downstream direction.

After manual calibration of the wave celerity (c) and the diffusion coefficient (D) the results obtained are comparable to the currently implemented simple routing scheme in terms of NS values. NS values are 0.80 and 0.72 for the stations Busti and Rasnalu respectively for the period 2000 – 2010. In general the average simulated discharge corresponds to the observed discharge. However, peak discharges are frequently underestimated, as well as the low flows during the months December – May.

The routing procedure is sensitive to changes in c and D, as well as to changes in the temporal and spatial resolution. Focussing on the latter two, it is not fully understood why a finer temporal and spatial resolution lead to substantial different results in simulated discharge for station Busti. Changes in c result in a slight shift in the arrival of a peak discharge as was expected. It was expected that higher values of D would lead to more attenuated waves and therefore lower peaks. However, the results of changes in D demonstrated the opposite. Further analysis is needed to explore this, especially focussing on the fact that mass is not preserved with changing parameter values.

Comparing computational time between the two routing schemes, it can be concluded that they are comparable. Depending on whether the user wants a visualisation of the results the cD-routing procedure takes 2 or 5 minutes longer than the currently implemented simple routing scheme.

It can be concluded that the newly implemented advanced cD-routing procedure performs well in terms of streamflow simulations and the results are comparable to the currently implemented simple routing scheme. Furthermore the cD-routing procedure has the advantage that it has the possibility to run the

routing on a different temporal and spatial resolution than the simulation of the SPHY model itself. It should be noted that currently the cD-routing procedure has only been applied to an area which is minimally regulated by humans and has substantial elevation differences. Further analyses should be done to test the routing procedure in different settings.

9. Recommendations

At this stage wave celerity (c) and diffusion coefficient (D) values were assumed to be constant over the whole research area. However, in reality these values will vary spatially depending on local channel geometry and characteristics. For instance lakes, reservoirs, channel contractions and structures like bridges, weirs and dams will influence these values. Making the c and D value spatial dependent could further improve the channel routing.

The sensitivity of the routing module to changes in c and D should be analysed further. This can be done by evaluating the effect of one parameter with the other parameter switched off. Furthermore, it would be interesting to take a closer look at a shorter time period to be able to analyse the response of the model to individual precipitation events and whether c and D can be calibrated to optimize the simulation for such individual events.

Calibration of the c and D values are done manually, but preferably this should be done in a consistent manner using a robust calibration method. Levenberg-Marquardt or HydroPSO are potential calibration methods to be used.

Adding the outflow from one reach to the next reach is done by injecting this as lateral inflow at one single point. However, another option that could still be implemented is to distribute this water along a transect of the next reach, rather than inject the whole volume of water at one point, because this might cause some disruptive effects on the routing in the next reach. These effects will become smaller when choosing a smaller threshold value.

Currently the length of the reaches are determined based on the number of cells of that reach multiplied with the length of one cell, i.e. the spatial resolution of the grid. This calculation is valid in the case when the water flows straight through the cell. However, water could also flow diagonally through a cell depending on the local drain direction, which would increase the flow path of the water. Taking this into account new calculations were performed based on the local drain direction (ldd) map, improving the estimation of the true length of the flow path that the water takes. However, the visualisation of the results were already programmed in such a way that it requires the length calculated based on the number of cells multiplied with the length of one cell. Due to time limitations it was not able to implement the improvement in calculations of the flow path length to the visualisation of the results and therefore the former, less accurate calculations were adopted again. Appendix B provides the programming code to calculate the length the water has to travel in every reach based on the ldd map.

Some pragmatic choices were made during the programming. For instance a reach is artificially elongated when it happens to consist of only one cell. This is needed for the calculations of the cD-equation among others. However, the effects of adding one cell is small in comparison with for instance the way the length of a reach is determined without considering the local drain direction.

Currently the downstream boundary condition is chosen to be the flow hydrograph of the accumulated runoff of the entire catchment. However, with this we force an amount of discharge at timestep t at the boundary that produced at that same timestep t, while in reality this runoff would arrive later at the outlet. Considering this, the flow hydrograph at the downstream boundary preferably should be shifted.

Currently there is no option available to include lakes and reservoirs in the routing procedure. Like

in HEC-RAS there is an option to include a storage area to simulate a reservoir or a dam breach (Brunner, 2010). Using the vector layer as is used now in this new routing procedure, a reservoir could become a point in the vector, for which this cell would be assigned different characteristics (e.g. a larger width and lower c) than a normal river cell. How this can be executed exactly needs to be explored further.

PCRaster itself also has the option to solve the dynamic wave equation, as well as the kinematic wave equation. However, the latter concerns an experimental implementation, and stability and accuracy cannot be guarenteed. Furthermore, to solve the dynamic wave equations substantial data and information is required as was concluded earlier as well (Sect. 4.1). In addition, working outside the PCRaster environment, as is done with the implementation of the cD-routing scheme, enables us to choose different temporal and spatial resolution used for the routing calculations.

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A. Overview python tables

Table A.1

Python table	Description	Format	File name
reachID	Locations of all cells belonging to a streamorder	x, y	reachid_*order*
ReachArrayID	Similar to *reachID* with slopelength (SL) and DEM value added. Sorted based on SL.	ID, x, y, DEM, SL	ReachArrayID_*order*
SubReachArrayID	Similar to *ReachArrayID* but now for each sub-reach within a streamorder. Sorted from up-to downstream	ID, x, y, SL, #order, #sub	SubReachArrayID_*order_sub*
outIDreach	Location of outlets of all reaches from all streamorders	#order, #sub, x, y	outIDreach
inIDreach	Location of inlets of all reaches from all streamorders	#order, #sub, x, y	inIDreach
upsIDreach	Location of inlets of all reaches with streamorder > 1	#order, #sub, x, y	upsIDreach
pitID	Same as outIDreach, however in the order that PCRaster scans the raster. Needed to convert arrays to maps again.	x, y	
inID	Similar to inIDreach, however in the order that PCRaster scans the raster and only reaches with streamorder 1. Needed to convert arrays to maps again.	x, y	
uphydroID	Contains the locations of inlets of reaches of streamorder > 1.	x, y	
inoutID	Contains the locations of all outlets plus inlets of reaches with streamorder 1.	x, y	
connectID	Location where a stream flows into the next stream. Order same as *pitID*.	x, y	connectID
coupled_reaches	Lists which reaches are connecting to each other. First two columns contain information about the upstream reach. The latter 2 columns store information about the reach it flows into.	#order, #sub, #order, #sub	coupled_reaches
TimeArrayPit	Stores for each outlet the accumulated runoff at that point for every timestep.	Timestep, outlet 1, outlet 2, ...	TimeArray_pits
TimeArrayIn	Stores for each inlet with streamorder 1 the accumulated runoff at that point for every timestep.	Timestep, inlet 1, inlet 2, ...	TimeArray_inlets
TimeArrayUphydro	Stores for each inlet with streamorder > 1 the accumulated runoff at that point for every timestep.	Timestep, inlet 1, inlet 2, ...	TimeArray_ups
TimeArrayOutlet	Stores for the outlet of the entire catchment the accumulated runoff for every timestep.	Timestep, inlet 1, inlet 2, ...	TimeArray_outlet
Uphydro	Hydrograph at the inlet of a reach.	#rows = #time steps	uphydro_*order_sub*
Downhydro	Hydrograph at the outlet of a reach.	#rows = #time steps	downhydro_*order_sub*
Lateral	Hydrograph at the cell that laterally drains into the next reach.	#rows = #time steps	lateral_*order**sub* *order_drain**order_sub*
Q_record_back	Stores for each cell of a reach the results of the cD-equation for each time step. '_back' refers to running the cD-equation in upstream direction.	#columns = #cells, #rows = #time steps	Q_record_back_*order_sub*
Q_record	Stores for each cell of a reach the results of the cD-equation for each time step. Now the cD-equation is applied in downstream direction.	#columns = #cells, #rows = #time steps	Q_record_*order_sub*
Qall	Stores for each cell of the entire stream network the results of the cD-equation for each time step. Used for visualizing the results.	#columns = #cells, #rows = #time steps	Qall.txt

B. Reach length based on ldd map

```
# Determine the length of the reach in km.  Based on the flow direction the flow path
# is either the spatial resolution of a cell (straight), or it flows diagonal
# over the cell.  In the latter case Pythagorean theorem is applied (diag).
# Find for each cell its drain direction
lddreach = pcr.ifthen(subreach,FlowDir_ini)
ldd_reacharray = pcr.pcr2numpy(lddreach, MV)

# Count the number of cells in which the water flows diagonally or straigth respectively.
diag = numpy.count_nonzero(ldd_reacharray==1) + numpy.count_nonzero(ldd_reacharray==3) +
numpy.count_nonzero(ldd_reacharray==7) + numpy.count_nonzero(ldd_reacharray==9)
straight = numpy.count_nonzero(ldd_reacharray==2) + numpy.count_nonzero(ldd_reacharray==4)
+ numpy.count_nonzero(ldd_reacharray==6) + numpy.count_nonzero(ldd_reacharray==8)
pit = numpy.count_nonzero(ldd_reacharray==5)

# Calculate the length of the reach.
length_reach = diag*(math.sqrt((SpaceRes**2)*2)) + (straight+pit)*SpaceRes
numpy.save(outpath + 'length_' + str(order) + '_' + str(sub+1), length_reach)
```

C. Calibration log-file

This appendix is written after the first results were presented already and it continues on the results that were found during the sensitivity analyses (i.e. it was found that mass is not preserved). Artificial examples were performed to test the numerics of the routing module. Apart from minor revisions the code appears to be solid. However, it was found that the routing module is highly sensitive to parameter values and parameter combinations. Therefore a script called '*cD_test.py*' was created in which many parameter combinations are tested and the preliminary results are written to a log-file. The user should specify the range for the parameters c, D, dt and dx in the config-file, as well as the range within which the simulated discharge should remain, otherwise a warning is raised.

```
12 # Length of reach in km
13 L = length

15 # Specify the weight given to Neumann and Dirichlet part of the Robin b.c.
16 alpha = config.getfloat('CDROUTING','alpha')
17 beta = config.getfloat('CDROUTING','beta')

19 # Implicit/explicit (omega=1 --> fully implicit, omega=0 --> fully explicit)
20 omega = config.getfloat('CDROUTING','omega')

22 # Generate TABLE to test/calibrate the results of several combinations
23 # of parameters c, D, dt, dx.
24 C_min = config.getfloat('CDROUTING','C_min')
25 C_max = config.getfloat('CDROUTING','C_max')
26 C_step = (C_max-C_min)/5 # 5 interval steps
27 C = np.arange(C_min,C_max+C_step,C_step)
28 if C_min == 0.0: # Cannot handle 0
29     C[0] = 0.0001

31 D_min = config.getfloat('CDROUTING','D_min')
32 D_max = config.getfloat('CDROUTING','D_max')
33 D_step = (D_max-D_min)/5 # 5 interval steps
34 D = np.arange(D_min,D_max+D_step,D_step)
35 if D_min == 0.0: # Cannot handle 0
36     D[0] = 0.0001

38 Dt_min = config.getfloat('CDROUTING','Dt_min')
39 Dt_max = config.getfloat('CDROUTING','Dt_max')
40 Dt_step = (Dt_max-Dt_min)/5 # 5 interval steps
41 DT = np.arange(Dt_min,Dt_max+Dt_step,Dt_step)

43 Dx_min = config.getfloat('CDROUTING','Dx_min')
44 Dx_max = config.getfloat('CDROUTING','Dx_max')
45 Dx_step = (Dx_max-Dx_min)/5 # 5 interval steps
46 DX = np.arange(Dx_min,Dx_max+Dx_step,Dx_step)
```

```
48 # Range within the simulated Q should stay, otherwise a warning is raised.
49 Q_max = config.getfloat('CDROUTING','Q_max')
50 Q_min = config.getfloat('CDROUTING','Q_min')

52 # Open/create log-file to write the results to.
53 f=open('logfile', 'w')
54 f.write("%s,%s,%s,%s,%s,%s,%s,%s,%s,%s,%s\n" %
    ('c','D','dt','dx','Courant','Peclet','Qmax','Qmin','Qtot_up','Qtot_down','WARNING'))

56 # Try all parameter combinations:
57 for c in C:
58     for d in D:
59         for dt in DT:
60             for dx in DX:
61                 warningFLAG = 0
62                 # Number of spatial steps
63                 Nx = int((L/dx)+1)
64                 x_grid = np.array([j*dx for j in range(Nx)])

66                 # Number of time steps
67                 Nt=50
68                 t_grid = np.array([n*dt for n in range(Nt)])

70                 # Define sigma and rho in the context of numerical discretization
71                 sigma = float(d*dt)/float(dx*dx)
72                 rho = float(-c*dt)/float(2.*dx)
73                 Courant = 4*-rho
74                 Peclet = dx*c/d

76                 # Create tridiagonal matrices A and B
77                 A = np.diagflat([(-sigma+rho)*omega for i in range(Nx-1)], -1) +
78                     np.diagflat([1.+2.*sigma*omega for i in range(Nx)]) +
79                     np.diagflat([(-(sigma+rho))*omega for i in range(Nx-1)], 1)
80                 A[0,:]  = np.array([1] + [0 for i in range(0,Nx-1)])
81                 A[-1,:]  = np.array([0 for i in range(0,Nx-3)] + [-1] +
                    [(2*beta*dx)/alpha] + [1])

83                 B = np.diagflat([(sigma-rho)*(1-omega) for i in range(Nx-1)], -1) +
84                     np.diagflat([1.-2.*sigma*(1-omega) for i in range(Nx)]) +
85                     np.diagflat([(sigma+rho)*(1-omega) for i in range(Nx-1)], 1)
86                 B[0,:]  = np.array([0 for i in range(0,Nx)])
87                 B[-1,:]  = np.array([0 for i in range(0,Nx-3)] + [-1] +
                    [(2*beta*dx)/alpha] + [1])

89                 # Create vector F
90                 f_vec = np.array([0 for i in range(0,Nx)])*dt

110                 # Specify initial condition
111                 base = config.getfloat('CDROUTING','Q')
112                 init = np.array([base for i in range(0,Nx)])
113                 Q = init

115                 # Impulse upstream
116                 impulsFLAG = config.getint('CDROUTING', 'impulsFLAG')
117                 if impulsFLAG == 0:
118                     value_impuls=50
```

```
119                         width_impuls = 2
120                         up = np.array(np.ndarray.tolist(np.linspace(base,value_impuls,width_impuls)
                            + np.ndarray.tolist(np.linspace(value_impuls,base,width_impuls))
                            + [base for i in range(0,Nt-(2*width_impuls))])
123                     else:
124                         amplitude = 50
126                         width_impuls = 5
127                         freq = 1.1
128                         x = np.arange(width_impuls)
129                         y = base+amplitude*np.sin(np.pi * freq * x / width_impuls)
131                         up = np.array(np.ndarray.tolist(y)+[base for i in range(0,Nt-width_impuls)]

135                     # Create empty list to store the results
136                     Q_record = []

138                     # First create a stationary solution which is subsequently provided
to
139                     # the main loop.  To create this stationarity no impulse is given
upstream.
140                     for ti in range(1,100):
141                         Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
142                         Q = Q_new
143                     Q_record.append(Q)

145                     # Main loop to solve the system iteratively
146                     for ti in range(1,Nt):
147                         # Provide upstream hydrograph
148                         f_vec[0] = up[ti]*dt
151                         # Solve the system
152                         Q_new = np.linalg.solve(A, B.dot(Q)+ f_vec)
153                         Q = Q_new
154                         if max(Q)>Q_max or min(Q)<Q_min:
155                             warningFLAG = 1
156                             print 'WARNING: large oscillations:
                             choose other parameter values'
157                             break
158                         Q_record.append(Q)

160                     # If warning is raised:  write the parameter values to the log-file
161                     # without continuing the calculations
162                     if warningFLAG==1:
163                         f.write("%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%s,%s,%s,%s,%s\n" %
                         (c,d,dt,dx,Courant,Peclet,'NaN','NaN','NaN','NaN','WARNING'))
164                         continue
165                     # If no warning is raised:  create array Q_record
166                     Q_record = np.array(Q_record)

168                     # Calculate the maximum and minimum simulated Q
169                     Qmax=[]
170                     Qmin=[]
171                     for time in range(0,Nt):
172                         Qmax.append(max(Q_record[time]))
173                         Qmin.append(min(Q_record[time]))
174                     Qmax = max(Qmax)
175                     Qmin = min(Qmin)
```

```
177                 # Calculate the total Q at the upstream and downstream end of the
reach.
178                 # If the wave is passed completely, these totals should be equal.
179                 Qtot_up = 0
180                 for j in range(len(Q_record)):
181                     Qtot_up+=Q_record[j][0]

183                 Qtot_down = 0
184                 for j in range(len(Q_record)):
185                     Qtot_down+=Q_record[j][-1]

192                 # Write the results of this parameter combination to the log-file
193                 f.write("%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%.2f,%s\n" %
                    (c,d,dt,dx,Courant,Peclet,Qmax,Qmin,Qtot_up,Qtot_down,'-'))
```